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THE CONSTRUCTION OF ALIGNMENT CHARTS

CONSTRUCTION OF ALIGNMENT CHARTS

BY

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PREFACE

There are comparatively few texts devoted to the principles underlying the construction of alignment charts. A large percentage of even this small number of texts give an exposition of the subject that is difficult of comprehension by the student of limited mathematical knowledge. This work is offered to those who wish to gain a workable knowledge of the subject but who are not interested in a highly specialized mathematical treatise. The presentation of the mathematics has, therefore, been kept as simple as possible. No reader who is familiar with the algebra, plane geometry, and the elements of trigonometry needs to fear going beyond his depth in mathematics. The subject matter of the text was first presented in the form of class notes, and it has been very gratifying to see the proficiency attained in the construction of alignment charts by students who have had no advanced mathematics.

While a large number of the equations encountered in engineering work fall under the cases discussed, it is recognized that the book is in the nature of an elementary text rather than an exhaustive reference. The attempt has been made to present in as simple a manner as possible a direct method of procedure in the construction of alignment charts for equations of various forms. A designer or engineer is seldom able to procure a chart which is entirely satisfactory for the particular problem at hand, and therefore, the ability to construct a chart for special conditions may enable him to save time and labor, and eliminate costly errors.

The equations used in the illustrative examples of the text and in the problems at the end of each chapter have been taken from engineering practice. Since it is the equations which are of

primary concern the significance of the variables and the units in which they are expressed have, in general, not been given. However, the engineer will recognize the symbols of the equations and appreciate that the limits imposed upon the variables are reasonable.

The charts of the text have been reduced in the process of reproduction and therefore will not check with the analytical work. However, in most cases one full size dimension has been given to enable the reader to determine the scale of the cut. The scales of the charts have not been completely graduated, and only such points have been shown as were considered necessary to illustrate the method of construction.

It is hoped that this text may be instrumental in promoting an interest in this useful and fascinating subject among students in schools, shops, offices, and drafting rooms.

G. W. SWETT

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THE CONSTRUCTION OF ALIGNMENT CHARTS

CHAPTER I

CONVERSION SCALES

THE increasing use of alignment charts in engineering texts, hand-books, and the technical press is evidence of the fact that designers and engineers now recognize the practical utility of this type of chart. Of the graphical methods in use to lessen the labor involved in the solution of formulas and equations, there are two which find the widest application and the greatest usefulness. These are the graphs or systems of curves (or straight lines) in which the relations of the several variables are shown on co-ordinate axes, and the alignment charts. The latter have certain distinct advantages in that the variables may be given wide ranges within a comparatively small space, and the open construction of the charts with satisfying values of the variables lying on straight lines makes for ease in reading and interpolation. Constructions requiring a large number of lines may take an undue amount of time to prepare and be difficult to read and interpolate, particularly when interpolations must be made between curves. The designer will find an alignment chart useful when tracing the effect of a change in the value of a variable in a preliminary calculation, and in working out and checking results for equations in frequent use. Whether or not a graphical chart for the solution of an equation will warrant the time spent in its construction will depend upon a number of factors; as (1)

the difficulty encountered in solving the equation, (2) the frequency with which the equation must be solved, (3) the time consumed in constructing the chart, (4) the ease with which the chart may be read, and (5) the accuracy obtainable. This text is intended to cover the principles underlying the construction of charts for the types of equations most often found in engineering practice, and those for which charts may be constructed without undue labor.

Scales. Since an alignment chart is merely the assemblage of a number of scales, only two factors need to be determined, viz., the proper location of the scales, and correct scale graduations. A general discussion of scales may properly be presented first. A scale may be defined as a straight or curved line carrying graduations. Curved scales are usually, but not necessarily, circular. The spacing of the graduations ordinarily follows a fixed law, which may be expressed in the form of an equation. Such "scale equations" are used in constructing the scales employed in making the charts and are not to be confused with the engineering equations for the solution of which the charts are made. These scales bear the same relation to an alignment chart that an architect's scale does to a scale drawing, the main difference lying in the fact that while drawings are made to a comparatively few uniform scales, alignment charts require a large number of scales many of which are not uniform, as will be pointed out later. Since a variable in an equation may be raised to any power, have any coefficient, appear as a logarithm, trigonometric function, etc., it is necessary to prepare scales to meet these conditions.

Uniform Scales.

The simplest case is that in which a variable appears as a first power, with or without a coefficient. Scale equations for this case are

$$x = n, \quad y = 3n, \quad z = 0.2n, \text{ etc.}$$

The left-hand member of such a scale equation (x, y, z, etc.) indicates the distance along the scale between the several values of the variable, as "n" in the given equations. It is evident that equal increments in n will be represented by equal distances along the scale, and the scale will be uniform. Also, x, y, and z can have neither coefficient nor exponent. Indicating values of the scale distance (x, y, z) on one side of the scale and corresponding values of n on the other side, the three equations indicated will give the scales shown in Fig. 1. It is to be noted that integral

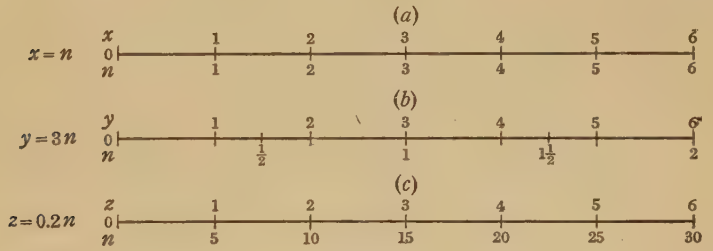


FIG. 1.

increments in the scale distances (x, y, z) will always be uniform in the construction of any scale, and the value of the variable, such as n, placed opposite a value of x is such that the corresponding values satisfy the scale equation.

NON-UNIFORM SCALES. (Variable raised to a power.)

When the variable in the equation for which the chart is to be constructed appears as a power, it is evident that the scale for the variable is no longer uniform. Taking first the cases where the exponent is positive, scale equations would be,

$$x = n^{\frac{1}{2}}, \quad x = n^2, \quad x = 0.5n^2.$$

These equations would be represented by the scales shown in Fig. 2. It is evident that

(1) For values of the variable (n) the scales are not uniform.

(2) The values of the variable (n) scale should be arranged

for ready interpolation irrespective of the x values, as, for example, in Fig. 2 (a) the values of n as shown are difficult to interpolate.

(3) Accuracy of interpolation varies widely with different values of the variable.

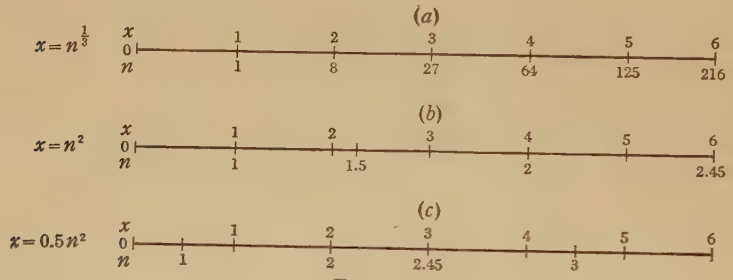


Fig. 2.

(4) Scale equations differing only by a coefficient of the variable have scale distances that are proportional to the coefficient. Note that the values of n in Fig. 2 (c) have one-half the spacings of those in Fig. 2 (b), the coefficients of n^2 bearing that ratio. The x scales, however, are identical. Compare also Fig. 1 (a, b, and c).

Scales for negative exponents can usually be avoided by changing the form of the chart equation. The special case of the reciprocal scale, viz., n^{-1} or $\frac{1}{n}$ is sometimes of use. This scale equation is

$$x = n^{-1} = \frac{1}{n}$$

and its scale is shown in Fig. 3.

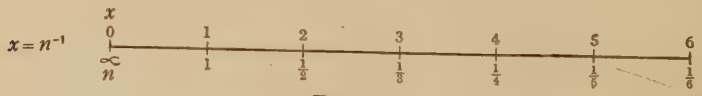


Fig. 3.

The disadvantage of this scale for any large integral variations in n is apparent, as the length of scale covering all values of n

from 1 to ∞ is equal to that covering values from $\frac{1}{2}$ to 1, and only one-half that covering values of n from $\frac{1}{3}$ to 1, etc. The usefulness of this scale is very limited.

NON-UNIFORM SCALES. (Variable a logarithm.)

The use of the "power" scales discussed above, with their attendant disadvantages, may often be avoided by the use of logarithmic equations. The logarithmic scales also have certain advantages over the seemingly more simple uniform scales. Logarithmic scale equations are

$$x = \log n, \quad x = 1.5 \log n, \text{ etc.}$$

The scales for these equations (only partially graduated) are shown in Fig. 4. Note that here again the scale distances for the *logarithm* of n are proportional to the coefficient.

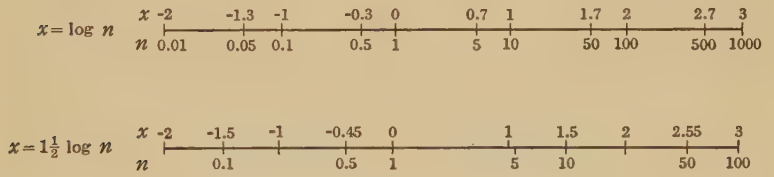


FIG. 4.

It is evident that a logarithmic scale cannot contain the value zero for the variable, as its logarithm is $-\infty$. Also, that the scale may include negative values of x even though the variable itself be positive since the logarithms of all numbers less than one are negative. In determining values of x for values of n less than unity, the decimal may best be written as a fraction and the logarithm found. Thus, for $n = 0.24$,

$$x = \log n = \log \tfrac{24}{100} = \log 24 - \log 100 = 0.38 - 2 = -1.62.$$

This value is not to be confused with the logarithm $\bar{1}.62$ where only the characteristic is negative.

When a scale must include wide limits for the variable, the logarithmic scale permits of an accuracy of practically the same number of significant figures throughout the range, since scale lengths for values 1 to 10, 10 to 100, 100 to 1000, etc. are equal.

NON-UNIFORM SCALES. (Trigonometric functions.)

The scale of a chart for the solution of an equation involving the trigonometric function of an angle should carry graduations in terms of the angle. As an example, Fig. 5 shows the scale for scale equation

$$x = \sin \theta.$$

Here again the values of θ should be so given as to permit of ready interpolation, irrespective of the values of x.

FIG. 5.

NON-UNIFORM SCALES. (Complex terms.)

A variable may appear in an equation in a more or less complex form. In this case a scale may be constructed for the variable, but each desired point of graduation must be determined independently. Such a scale equation is

$$x = \log \frac{d^5}{1 + \frac{3.6}{d}}.$$

The scale is graduated by assigning any number of desired values to the variable d, and determining the corresponding values of x. A table of corresponding values may then be made, as follows:

d	$\frac{1}{2}$	1	2	4	10	12	1.3
$\frac{d^5}{1+\frac{3.6}{d}}$	0.06	0.47	3.38	23.2	271	438	1
x	−1.22	−0.33	0.53	1.37	2.43	2.64	0

The partially graduated scale is shown in Fig. 6.

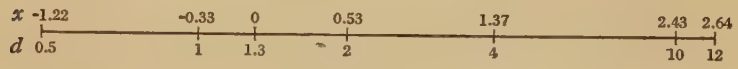


FIG. 6.

Scale Modulus or Scale Factor. While the x scales so far shown have been uniform, as is necessarily the case, no particular unit of length has been used. In designing a chart it will be necessary so to choose this unit of length that a given range of a variable may be shown on a scale of a desired length. It is evident that the larger the chart, other conditions remaining constant, the greater will be the accuracy of the results. The *scale modulus* or scale factor may be defined as that length of scale in inches, or any other desired unit of length, which is necessary to show unit increase in the variable or function of the variable. Throughout this text scale moduli will be expressed in inches and the letter "M" will be used as the symbol. A subscript indicates that the scale modulus is that used for the scale of the subscript letter, as "M_x" is the scale modulus for the scale of x. For a range of x of from 5 to 13, and $M_x = 1$, a scale 8 inches long will be needed. Or, given $M_{\log n} = 10$, it will require a scale 10 inches long to show values of n from 10 to 100, there being an increase of unity in log n for this range of n. In general, *the length of scale is equal to the scale modulus multiplied by the range of the variable or function of the variable*, viz.,

$$\text{scale length} = \text{modulus} \times \text{range}.$$

It is to be noted that in the cases of the power scales, log scales, trigonometric function scales, etc., that while the scale shows values of the variable itself, as "n" or "θ," the scale modulus refers to the values of n^3, log n, cos θ, etc., as the case may be. In the case of the logarithmic scale, for example, the scale modulus shows unit increase in the value log n, although the scale shows "n" directly.

As has already been pointed out the effect of a coefficient is to proportionately increase or decrease the length of scale between corresponding values of the variable, therefore a coefficient will have the same effect on a scale modulus. To illustrate, if, in Fig. 2 (*b*) (page 4) $M_x = 1$, the upper, or x scale, is an ordinary rule 6 inches long, then $M_{n^2} = 1$ also, and as 1 inch shows unit increase in n^2, it requires 6 inches to show an increase in n^2 of 6, or in n of 2.45, as from $n = 0$ to $n = 2.45$. The n scale is graduated in terms of n, *not* n^2. In Fig. 2 (*c*) the scale distances of the values of n^2, and hence of the scale values of n, are one-half those in Fig. 2 (*b*), the coefficient of n^2 being 0.5. Thus for $M_x = 1$, M_{n^2} is equal to 0.5 and, in general, in laying down the scale for the variable, use a scale modulus equal to the scale modulus for the x scale multiplied by the coefficient of the variable or of the function of the variable. For the scale equation, $x = 2.3 \log n$, and $M_x = 3$, the scale modulus for the n scale will be

$$M_{\log n} = 2.3 \times 3 = 6.9.$$

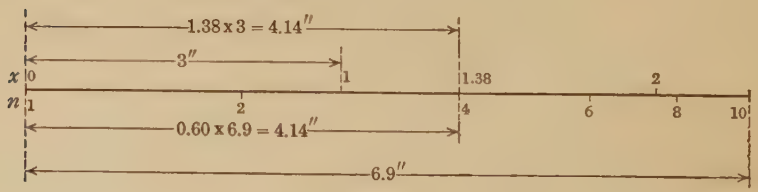


FIG. 7.

Figure 7 shows the relation of the x and n scales, the x scale graduated for $M_x = 3$ and the n scale graduated for $M_{\log n} = 6.9$.

The x scale is, of course, uniform while the values shown on the n scale are spaced proportional to the logarithms of the numbers, with 6.9″ showing an increase of n from 1 to 10, or unit increase in the logarithm of n. As a check note that since $x = 2.3 \log n$, for $n = 4$,

$$x = 2.3 \times 0.6 = 1.38.$$

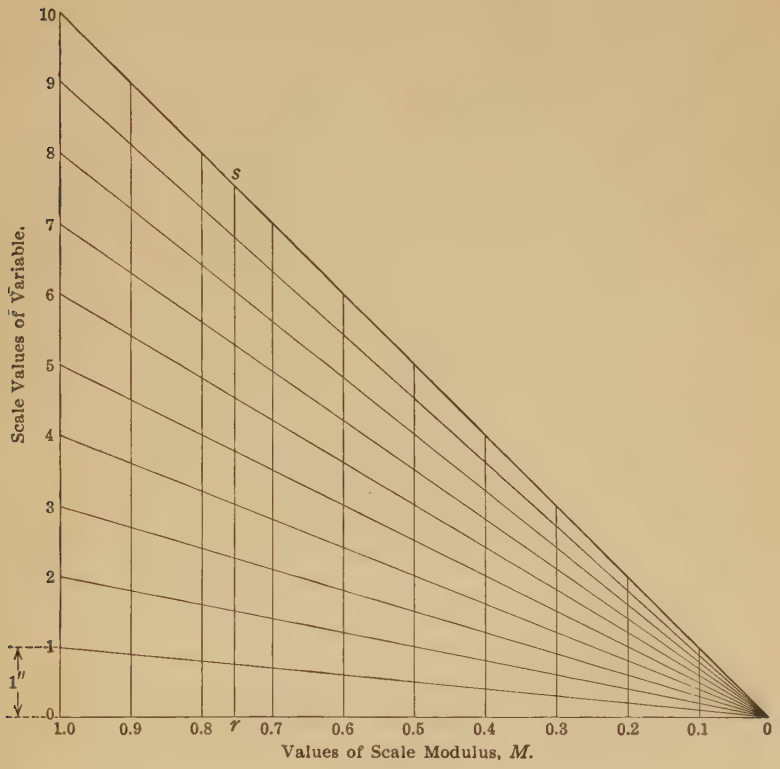


FIG. 8

The x distance to $n = 4$ is then

$$3 \times 1.38 = 4.14''.$$

On the n scale the distance is $\log 4 \times 6.9$ or $0.60 \times 6.9 = 4.14''$.

From the above it follows that in laying down the scale for a

variable having a coefficient the scale modulus to be used is equal to the scale modulus of x in the scale equation multiplied by the coefficient.

Charts to be used in laying down a scale having a required

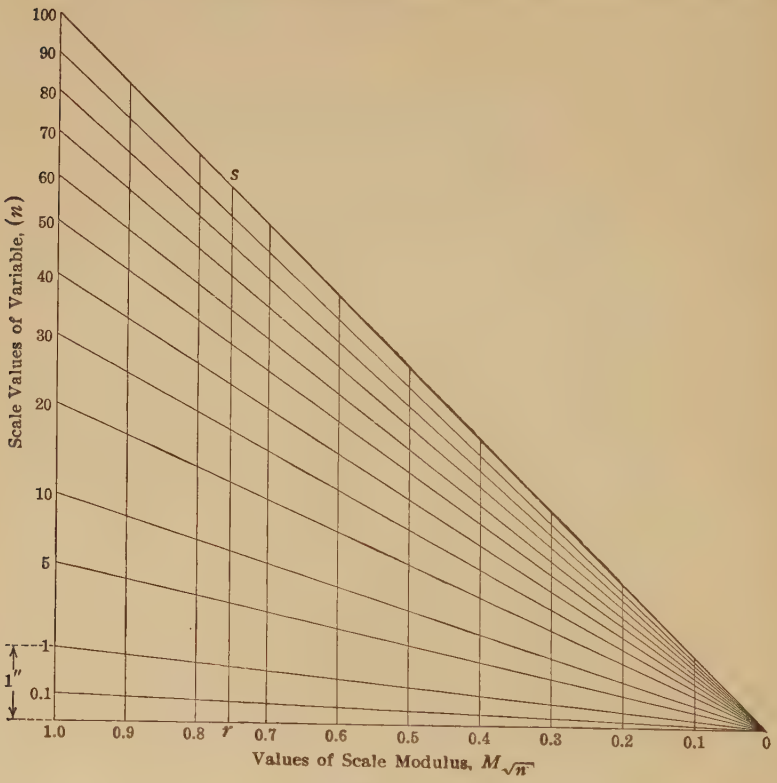


FIG. 9.

modulus may be prepared on a good grade of thin paper. Folding the paper on a desired modulus provides the scale for transferring the graduations. Figures 8, 9, 10, and 11 show such charts for the functions of the variables most frequently met.*

* Charts shown in Figs. 8, 9, and 10 may be purchased (John Wiley & Sons, Inc.) very reasonably and will be found of great assistance in chart construction.

In the cases of functions of variables occurring less frequently the scale may be constructed directly on the alignment chart.

Figure 8 shows the chart to be used in graduating uniform scales. Fold on the line *rs* (7.6 inches long) to graduate a scale having

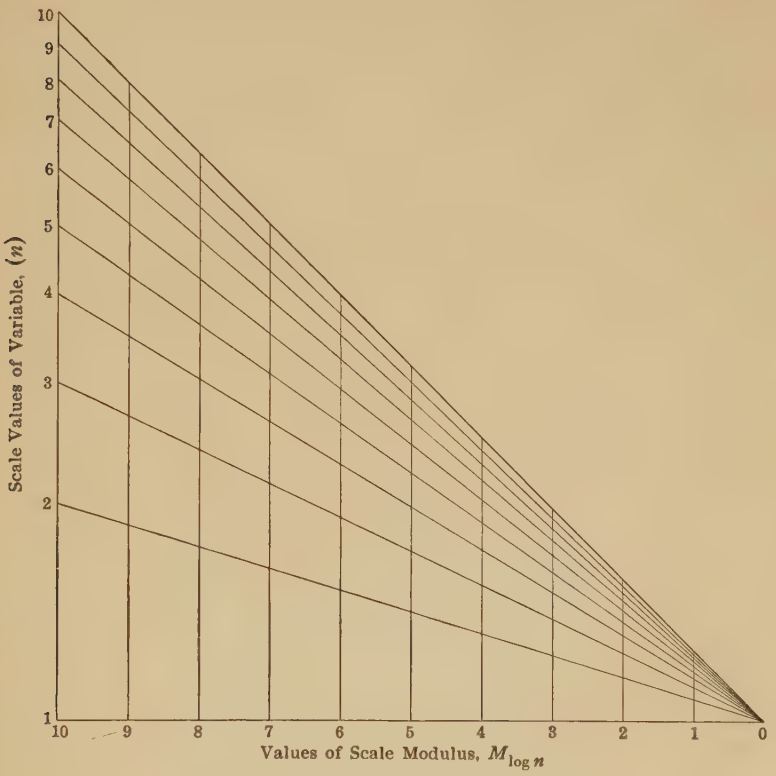


FIG. 10.

a modulus (M) of 0.76. For values of M one-tenth of those shown, multiply the scale values by 10, viz., for $M = 0.076$, scale values shown are 0, 10, 20, etc. An engineer's scale may often be used to advantage. Thus, to graduate a scale having a modulus of 0.25 ($M = 0.25$) the divisions on the "40 scale" are the correct ones to use.

Figure 9 shows the chart for square root scales. This chart may be used for other scale moduli by multiplying or dividing the scale values by ten. If it is desired to use scale graduations one-tenth of those shown, multiply the given values of M by

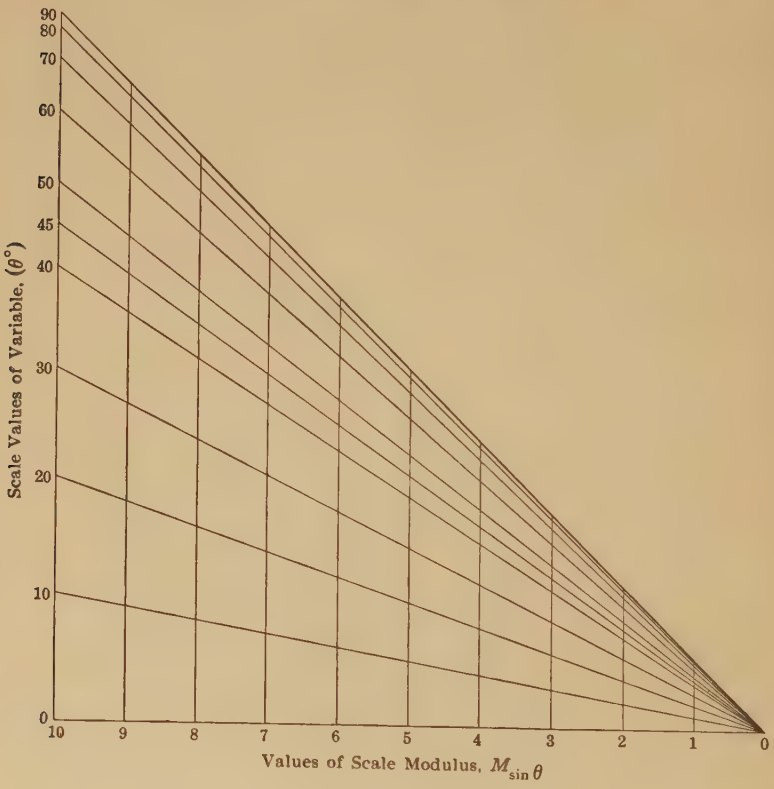


FIG. 11.

3.16 (viz., $\sqrt{10}$). Thus, the scale *rs* on scale modulus 0.76 may also be used for $M = 0.76 \times 3.16 = 2.40$, the scale values shown, then becoming 0.01, 0.1, 0.5, 1, 2, 3, etc.

Figure 10 shows the chart for logarithmic scales, the left-hand scale being 10 inches long and carrying scale values from 1 to 10.

(Note that a logarithmic scale can never carry the value zero.) For a given modulus the scale values shown may be multiplied or divided by 10, 100, 1000, etc. Scales for values of moduli greater than 10 may be graduated by noting that the scale distances are proportional to the values of $M_{\log n}$, thus a scale for $M_{\log n} = 15$ may be graduated by doubling the distances between scale values shown for $M_{\log n} = 7.5$.

Figure 11 shows a chart for graduating a sine function. The scale values read in degrees. The scale length for $M_{\sin \theta} = 10$ is 10 inches long showing values of θ from 0° to 90°, there being unit increase in the sine for the 90° increase in θ.

Equations of Two Variables. The construction of alignment charts for various types of equations will now be taken up, the two-variable equation being considered first.

Charts may be constructed for equations containing two variables which can be put into any one of the following forms:

1. $x = y$
2. $x = -y$
3. $x = y + k$
4. $x = -y + k$

where x stands for any term having only one variable. This variable may have any power and any coefficient or it may appear in a more complex form (see page 6). Similarly y represents a term of one variable to any power and having any coefficient. The value k is a constant, either positive or negative. Values of the variables are carried on either side of a single scale.

FIG. 12.

It will be shown that M_x must equal M_y if values which satisfy the equation, $x = y$, are to appear opposite one another on the scale.

In Fig. 12 assume the x scale to lie to the left and the y scale to the right of the line. If $x = y$ (Form 1 above) when $x = 0$, $y = 0$ and these points are opposite on the scale.

Assume "a" and "b," specific values for x and y respectively, which satisfy the equation.

Then $$a = b.$$

The number of inches of scale from the value $x = 0$ to the value $x = a$ will be $M_x a$, since M_x is the increase per unit and there are "a" units.

Since the values "a" and "b" satisfy the equation, point "b" on the y scale must lie opposite point "a" on the x scale. Hence the distance from point $y = 0$ to point $y = b$, must be $M_x a$, but this distance is also $M_y b$, therefore

$$M_x a = M_y b$$

and $$M_x = M_y$$

since $$a = b.$$

The effect of a minus sign before a term as in Form 2, viz., $x = -y$, is to invert the scale of the variable. Figure 13 shows the scales for this case, corresponding values of x and y satisfying the equation when the y scale is inverted.

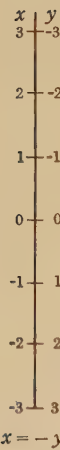


Fig. 13.

The only effect of a constant, as k in Forms 3 and 4, is to change the relative position of the two scales. This may be shown by letting $y' = y + k$ in Form 3.

Whence

$$x = y + k = y'$$

or $$x = y'$$

as in Form 1.

Fig. 14.

The values on the y' scale may be made to read in terms of y by subtracting the value of the constant k, since $y = y' - k$. This is shown in Fig. 14 for the points 0 and k, for the three scales x, y', and y. It is evident that other points would bear the same relation, and that the scales are completely determined when M_x and M_y are

known, and any one pair of values of x and y which satisfy the equation.

The method of procedure in laying out charts for two variables, or "conversion" charts as they may be called, will be shown by two examples.

Example 1. Prepare a conversion chart for changing inches to hundredths of a foot. The scale is to include values from 0 inches to 36 inches.

Letting n = number of inches,
and h = number of hundredths of a foot,

the relation between them is expressed by the equation

$$\frac{n}{12} = \frac{h}{100},$$

hence

$$n = 0.12h.$$

This equation is in the form $x = y$, where $x = n$, and $y = 0.12h$.

Since n is to vary from 0 to 36,
x will also vary from 0 to 36.

The range of the variable being 36, a scale length of 9 inches will require a scale modulus of

$$M_x = \tfrac{9}{36} \quad \text{or} \quad \tfrac{1}{4} \text{ inch},$$

since, length of scale = scale modulus × range, (see page 7)

or

$$\text{modulus} = \frac{\text{scale length}}{\text{range}}.$$

Then

$$M_n = M_x = \tfrac{1}{4},$$

the coefficient of n being one. (See page 8.)

The scale modulus for y being equal to that for x,

$$M_x = M_y$$

or

$$M_y = \tfrac{1}{4};$$

but the coefficient for h is 0.12, and therefore the scale modulus for graduating the h scale will be

$$M_h = 0.12\, M_y = 0.12 \times \tfrac{1}{4} = 0.03.$$

There being no constant, and the zero point being included in the scale, it may be used as a starting point.

Figure 15 shows the completed chart.

To graduate the "n" scale note that M_n is 0.25, that is 0.25 inch length of scale will show unit increase in the value of n. Consequently 2.5 inches will show an increase of n of 10 units. Fold the uniform scale (see Fig. 8) along the parallel through 0.25 and mark off the divisions. This will give the correct graduations for the n scale. The scale may also be graduated by using the fortieth's scale on an engineer's scale, as this will divide $2\frac{1}{2}$ inches into 100 divisions. Each division will then be 0.1 n.

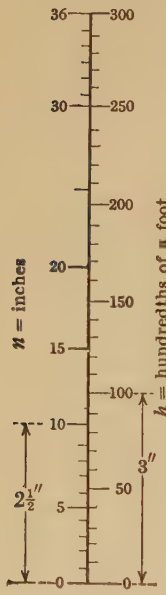


Fig. 15

The "h" scale having a modulus (M_h) equal to 0.03 is graduated by folding the uniform scale through 0.3 and multiplying the values by 10. Since 0.03 inch shows an increase of unity in the value by h, it is evident that 3 inches will show an increase of 100.

Example 2. Prepare a chart for the two-variable equation

$$pv^{1.5} = 500.$$

Scale length to be made approximately 10 inches and p to vary from 1 to 200.

This equation may be put into the form $x = -y + k$ by taking logarithms.

Thus $\log p = -1.5 \log v + \log 500$

where $x = \log p$

$y = 1.5 \log v$

and $k = \log 500.$

Since p varies from 1 to 200
$\log p$ varies from 0 to 2.3

and $x = \log p$ therefore varies from 0 to 2.3 or a range of 2.3 units.

From equation,

$$\text{Length of scale} = \text{scale modulus} \times \text{range}$$
$$10 = M_x \times 2.3$$

or
$$M_x = 4.35.$$

For convenience use

$$M_x = 4.5$$

giving a scale length of $4.5 \times 2.3 = 10.35''$.

Since the coefficient of $\log p$ is unity

$$M_{\log p} = M_x = 4.5.$$

From $M_x = M_y$, the scale modulus for $\log v$ may be found.

Thus $M_y = 4.5$

and $M_{\log v} = 1.5 \times 4.5 = 6.75$

the coefficient of $\log v$ being 1.5.

A starting point for the v scale is best found by assuming a value for p within the specified range and solving the equation for the corresponding value of v, thus for $p = 1$

$$1 \times v^{1.5} = 500$$
$$v = 63.$$

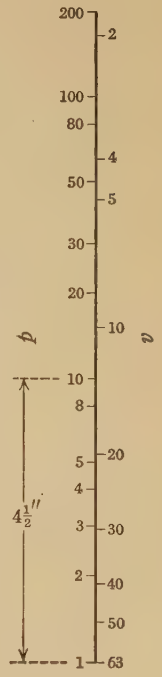


FIG. 16.

The p scale is laid off from the logarithmic scale by folding along a parallel through 4.5 since $M_{\log p} = 4.5$. (See Fig. 10.)

The v scale is inverted since the $\log v$ term has a minus sign and is laid off by folding on a parallel through 6.75 of the loga-

rithmic scales. The scale is located by placing the 63 value opposite the value 1 on the p scale, since when

$$p = 1, \quad v = 63.$$

Figure 16 shows the chart (only partially graduated).

PROBLEMS

1. Given $x = 2.4 \log T$

T to vary from 0.3 to 30.

What M_x will give a scale 9.6 inches long?
What $M_{\log T}$ would be used for this scale?

2. Given the scale equation

$$y = 2.45 \log r$$

(*a*) If r varies from 0.5 to 3.5 in a scale length of 8 inches, what is the value of M_y?
(*b*) If $M_y = 2.5$ and the minimum value of r is 1, what is the maximum value of r on a scale 10 inches long?
(*c*) Given $M_y = 2$, what is $M_{\log r}$?

3. Given the scale equation

$$z = 3r^2$$

(*a*) If $M_z = \frac{1}{30}$ and the minimum value of r is zero, what is the maximum value of r on a scale 8.10 inches long?
(*b*) If r is to vary from 5 to 10 in a scale length of 10 inches, what is M_{r^2}?

4. Given the scale equation

$$x = 8\sqrt{H}$$

H to vary from 0 to 50.

Assuming $M_x = 0.20$, what is the length of scale?
What $M_{\sqrt{H}}$ should be used? How many inches from the zero end of the scale is the point for graduation $H = 5$?

5. Prepare the scale for equation

$$x = \tan \beta$$

β to vary from 15° to 60°.

Use $M_x = 6$.

6. Prepare the scale for equation

$$y = 0.20(\sqrt{L} + \tfrac{1}{8}\sqrt{L}).$$

L to vary from 0 to 100, in a scale length of 9 inches.

7. Prepare a scale for equation

$$z = \tfrac{10}{3}(A - 0.6\sqrt{A})$$

A to vary from 0 to 25, in a scale length of 11 inches.

8. Prepare a scale for the equation

$$y = \log\left(\frac{n^2 + 1}{n^2 - 1}\right)$$

n to vary from 1.1 to 5, and $M_y = 5$.

9. Write an equation expressing the relation between cubic feet per second (n) and millions of gallons in 24 hours (h).

Prepare conversion charts as follows:

(a) Uniform scale (n to vary from 0 to 500).

(b) Logarithmic scale (n to vary from 2 to 500).

Note. — In problems 10 to 23 inclusive use logarithmic scales unless otherwise specified.

10. Given $T = 35{,}500\,F^{0.6}$

$T_1 = 1640\,F^{0.7}$

where T = drill thrust in pounds, T_1 = drill torque in ft. lbs. } for 1 inch drill

F = feed in inches per revolution of spindle.

Prepare a chart for each of the above equations making F vary from 0.002 to 0.2.

11. Given Net HP. $= 2\sqrt[3]{(\text{Gross HP.})^2}$.

Prepare a chart for gross horsepowers 25 to 500.

12. Prepare a chart for the equation

$$t = \frac{\pi l^{\frac{1}{2}}}{\sqrt{386}}$$

where t = time of vibration of pendulum in seconds

l = length of pendulum in inches

l to vary from 2 inches to 72 inches.

13. Prepare a chart for the equation

$$pv^{1.065} = 483$$

p to vary from 1 to 200.

14. Prepare a conversion chart representing the equation

$$T = 200\,F^{0.8}.$$

F to vary from 0.2 to 6.3.

Use a scale length of $9'' \pm 0.1''$.
Show the following values:

F, 0.2, 0.5, 0.7, 0.8, 1, 2, 2.5, 4.5, 6.3
T, 60, 100, 200, 300, 500, 700, 800.

15. Given the equation

$$N = 2\sqrt[3]{G^2}.$$

Prepare a chart 9 inches long covering a range of N from 10 to 1000.
Show values of N, 10, 25, 450, 650, 1000.
Show values of G, 65, 500, 1100.

16. Construct a conversion chart for the equation

$$D = 0.75d.$$

d to vary from $\frac{1}{2}$ to $3\frac{1}{2}$.

Use uniform scales.

17. Construct a conversion chart for the equation

$$\sqrt{d} = \frac{1.57}{P}$$

P to vary from 1 to 10.

Show only integral values for P.

18. Make a conversion chart for the equation

$$7500d^2 = L$$

d to vary from $\frac{1}{4}$ to 2.5.

Show values

d, $\frac{1}{4}$, $\frac{1}{2}$, 1, 1.3, 1.7, 2, 2.5
L, 500, 5000, 7500, 15,000, 20,000, 30,000.

19. Construct a conversion chart for the equation

$$A = 0.00545D^2$$

D to vary from 10 to 100.

20. Construct a conversion chart for the equation

$$h = 1.6P$$

h to vary from 0 to 72.

Use uniform scales 9 inches long.
(*Suggestion.* — Use engineer's scale.)

21. Make a conversion chart showing corresponding values of "head of water" and pressure (lbs. per sq. in.) using equation

$$P = 0.434H.$$

P to vary from 0 to 100.

Use uniform scales.

Show the following values of P and H:

P, 0, 10, 20, 50, 75 and 100.

H, 0, 50, 100, 125, 200, 250.

22. Prepare a chart for converting "head" into velocity using equation

$$v = 8\sqrt{h}.$$

h varies from 1 to 100.

Show values of h, 1, 5, 30, 75 and 100. Show values of v, 8, 20, 50 and 75.

23. Prepare a conversion chart for the equation

$$\lambda = \frac{300{,}000}{f}$$

λ to vary from 200 to 600.

CHAPTER II

EQUATIONS OF THREE VARIABLES (PARALLEL SCALES)

ALIGNMENT CHARTS for equations involving three variables may be prepared using three parallel scales so located and graduated that a straight index line will cut the scales at values of the variables which will satisfy the given equation. For such charts the equations must be put in the form

$$x + y = z$$

or

$$x + y = z + k.$$

The charts for these two equations are identical except that the graduations on the Z scale are moved as a unit an amount depending upon the value of the constant, k. Any term may have a negative sign. Limits may be imposed upon two of the variables, preferably x and y.

The chart is of the three parallel scale type. The left-hand scale is the X scale, the right hand, the Y scale and the Z scale is located between the other two.

The chart is read by placing a straight line (index line) through known values of any two of the variables. The intersection of the straight line with the scale of the third variable will give the value of that variable which will satisfy the equation.

Limits having been imposed upon values of x and y, the values of M_x and M_y may be determined from the range and assumed scale lengths. It is then necessary to find M_z in terms of M_x and M_y, and also the ratio of the distances between the scales. The X and Y scales may be placed at any distance apart, but for accuracy the intersections of the index lines and scales should not be too flat.

Determination of Scale Locations and Scale Moduli for Three Parallel Scale Charts

Given equation of the form

$$x + y = z.$$

Scale moduli M_x and M_y being known, it is desired to find M_z and the relative position of the Z scale in terms of M_x and M_y.

Assume the vertical scales as shown in Fig. 17, the Z scale being located at distances r and s from the X and Y scales, respectively.

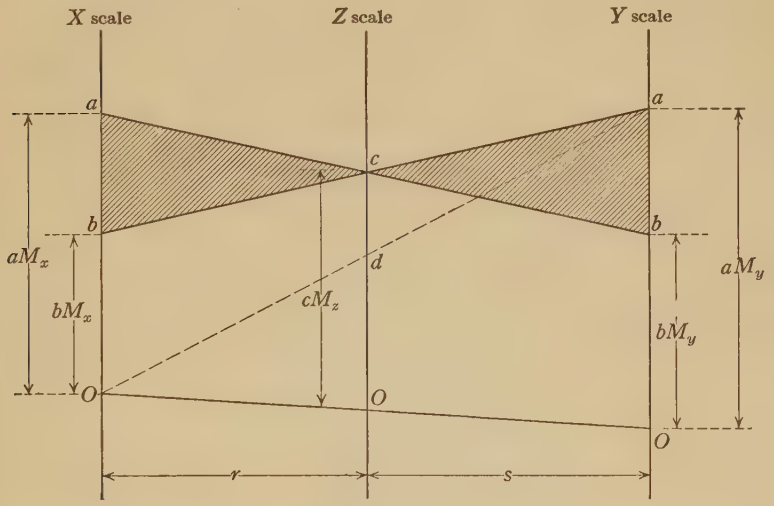


Fig. 17.

The zero point on the Z scale is located on the straight line joining value $x = 0$ with $y = 0$, since for these values of x and y, z must be equal to zero.

Let a, b, and c be specific values of x, y, and z respectively which satisfy the equation, then

$$a + b = c.$$

The length (in inches) of X scale from $x = 0$ to $x = a$ will be aM_x. Similarly on the Y scale to point $y = b$, the length will be bM_y.

Since the values a, b, and c satisfy the equation a straight line joining point "a" (X scale) to point "b" (Y scale) must cut the Z scale at point "c," a distance cM_z from the zero point.

Now assume

$$x = b \text{ and } y = a,$$

then

$$x + y = b + a$$

and z must equal c as before.

A line joining "b" (X scale) to "a" (Y scale) must cut the Z scale at point "c" as before, since there has been no change in the value of z.

From similar triangles shown shaded

$$\frac{aM_x - bM_x}{aM_y - bM_y} = \frac{r}{s}$$

or

$$\frac{M_x}{M_y} = \frac{r}{s} \tag{1}$$

which locates the Z scale with reference to the X and Y scales.

Draw the line $x = 0$, $y = a$ (shown dotted).

Then

$$\begin{aligned} cM_z &= cd + d0 \\ &= bM_x\left(\frac{s}{r+s}\right) + aM_y\left(\frac{r}{r+s}\right) \\ &= bM_x\left(\frac{M_y}{M_x + M_y}\right) + aM_y\left(\frac{M_x}{M_x + M_y}\right) \\ cM_z &= (a+b)\left(\frac{M_xM_y}{M_x + M_y}\right) \end{aligned}$$

or

$$M_z = \frac{M_xM_y}{M_x + M_y} \tag{2}$$

since

$$a + b = c.$$

An example will show the method of procedure and the use of equations (1) and (2).

Example 3. Prepare a three parallel scale chart for the equation

$$P = 9400\frac{d^3}{r}$$

d to vary from 0.10 to 0.65
r to vary from 0.50 to 2.00.

Scale lengths to be 6 inches, approximately.

The equation is placed in the proper form by taking logarithms. Thus

$$\log P = 3 \log d - \log r + \log 9400$$

or
$$3 \log d - \log r = \log P \mp \log 9400$$

where
$$x = 3 \log d$$
$$y = -\log r$$
$$z = \log P$$
$$k = \log 9400.$$

Since d varies from 0.10 to 0.65
$\log d$ varies from -1 to -0.187.

The logarithm of 0.65 may be obtained by writing,

$$\log \left(\tfrac{65}{100}\right) = \log 65 - \log 100 = 1.813 - 2.00 = -0.187.$$

Then, $x = 3 \log d$ varies from -3 to -0.561

or a range for x of 2.44 units (nearly).
For a scale 6 inches long

$$M_x = \frac{6}{2.44} = 2.46.$$

For convenience take

$$M_x = 2.5$$

giving a slightly longer scale.

The coefficient of log d being 3

$$M_{\log d} = 3M_x = 7.5.$$

To find M_y and $M_{\log r}$, note that

$$\begin{aligned} r &\text{ varies from } 0.5 \text{ to } 2. \\ \log r &\text{ varies from } -0.3 \text{ to } 0.3 \\ y = \log r &\text{ varies from } -0.3 \text{ to } 0.3 \end{aligned}$$

or a range for y of 0.6 units.

For a 6-inch scale

$$M_y = \frac{6}{0.6} = 10.$$

Therefore $$M_{\log r} = M_y = 10$$

the coefficient of log r being unity.

Assuming the X and Y scales any convenient distance apart (Fig. 18), the Z scale may be located from equation (1) (page 24).

$$\frac{r}{s} = \frac{M_x}{M_y} = \frac{2.5}{10} = \frac{1}{4}.$$

Thus the Z scale is placed at a distance from the X scale equal to one-fifth of the assumed distance between X and Y. This may be done conveniently by assuming five units of scale length at least equal to the distance between X and Y, and, placing the ends on these scales, mark the point at the end of the first unit from scale X. This locates the Z scale.

Note that the Z scale is located by the scale moduli of x and y, *not* from the moduli of log d and log r.

The scale modulus M_z is obtained from equation (2) (page 24).

$$M_z = \frac{2.5 \times 10}{2.5 + 10} = 2,$$

therefore, $$M_{\log P} = 2.$$

A starting point for the P (Z) scale is found by assuming values of 0.5 and 1.00 for d and r respectively, and finding a value for P from the given equation.

$$P = 9400 \times \frac{(0.5)^3}{1} = 1175.$$

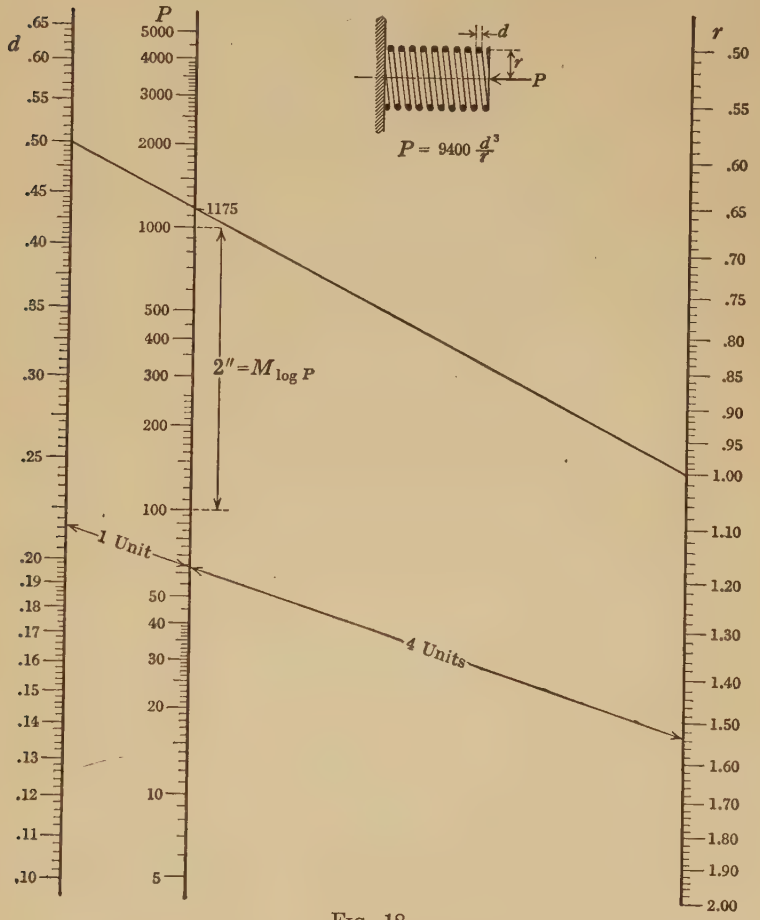


FIG. 18.

To graduate d fold on logarithmic scale 7.5, for scale r fold on 10, and for the P scale fold on 2 placing 1175 at the intersection

of the Z scale and the index line connecting $d = 0.5$ and $r = 1.00$. The r (Y) scale is inverted since it has a negative sign. (See Fig. 13.)

Figure 18 shows the completed chart.

Equations of three variables frequently include a constant term as in

$$t = \frac{pr}{11{,}000} + \tfrac{1}{4}$$

and

$$D = 1.24\sqrt{\frac{L}{f}} + 0.088.$$

These equations may be placed under the case just discussed by substituting a new variable which will differ from one of the variables of the equation by an amount equal to the constant. Thus, in the first of the equations cited above t' may be substituted for $t - \frac{1}{4}$ and the equation written $t' = \frac{pr}{11{,}000}$. By taking logarithms the equation may be put in the proper form and a chart constructed. This chart will show a t' scale. This scale may be made to read t directly by increasing the graduation readings of the t' scale by $\frac{1}{4}$, thus values of $\frac{1}{2}$, $\frac{3}{4}$ and 1 of the t' scale would read $\frac{3}{4}$, 1 and $1\frac{1}{4}$ if the scale were to show values of t directly.

A similar treatment may be used when one or more of the variables appears in a somewhat more complicated form. For example, in the equation

$$f = p\left(\frac{n^2 + 1}{n^2 - 1}\right)$$

there are only three variables, and a three parallel scale chart may be made as follows. Since only the variable n appears in the parenthesis, substitute

$$r = \frac{n^2 + 1}{n^2 - 1},$$

when

$$f = pr.$$

The r scale of a chart prepared for this equation may be graduated in terms of n since r and n are dependent variables. (See Problem 8.) The scale of r will read directly in terms of n provided values of r as shown in the following table are replaced by corresponding values of n.

r	n	
10.5	1.1	
2.6	1.5	
1.67	2	
1.25	3	etc.

While the scales for this type of chart are usually logarithmic, an interesting exception is found in the chart for the equation

$$\frac{T_1}{T_2} = e^{0.3\alpha}$$

or

$$\frac{T_1}{T_2} = e^{0.0052\alpha_1},$$

where α is expressed in radians, and α_1 in degrees.

Taking logarithms and transposing

$$\log T_1 - 0.0052\,\alpha_1 \log e = \log T_2$$

or, since $\log e = 0.4343$

$$\log T_1 - 0.00226\,\alpha_1 = \log T_2.$$

It is thus seen that while the T_1 and T_2 scales are logarithmic, the α_1 scale is uniform.

PROBLEMS

24. Prepare a three parallel scale alignment chart for the equation

$$V = 0.785D^2H$$

where V = volume of a cylinder of diameter D, and length, H.

D to vary from 1 to 10
H to vary from 1 to 20.

25. Prepare a three parallel scale chart for the equation

$$S = \frac{bh^2}{6}$$

b to vary from 1 to 10
h to vary from 1 to 8.

Scales to be not over 10 inches long or less than 9 inches.
Show index line in position to solve

$$b = 2\tfrac{1}{2}, \qquad h = 6\tfrac{1}{2}, \qquad S = ?$$

26. Prepare a chart representing the equation.

$$5l + 3r - 4t = 30$$

l to vary from -10 to 10
r to vary from -40 to 60.

Scale lengths to be 10 inches.
Show integral values of l, and values of r and t in multiples of 10.
Show an index line for $l = -3$, $r = 40$, $t = ?$

27. Construct an alignment chart for the equation

$$W = 2.7d^2l$$

d to vary from 2 to 16
l to vary from 1 to 25.

28. Construct an alignment chart for the equation

$$f = \frac{T}{0.2d^3}$$

d to vary from 4 to 16
f to vary from 2000 to 10,000.

29. Construct an alignment chart for the equation

$$P = 4500\,\frac{d^{1.65}}{N^{0.40}}$$

P = load (per ball) on a ball thrust bearing in lbs.
d = ball diameter. Varies from $\frac{1}{8}$ to $1\frac{1}{4}$.
N = R.P.M. Varies from 50 to 500.

30. Make a three parallel scale alignment chart for the equation

$$w = 75{,}000\,\frac{t^{2.21}}{d^{1.16}}$$

t to vary from 0.05 to 0.3
d to vary from 2 to 30.

Scale lengths to be approximately 7 inches.

31. Prepare an alignment chart for the equation $P^4 = \frac{14{,}340t^3}{p}$

where P = Pitch of bolts in flange in inches
p = pressure in pounds per square inch
t = thickness of flange in inches.
P to vary from 1 to 8
p to vary from 200 to 2000.

Show four values of each variable, and an index line in position to solve $P = 4, p = 1250, t = ?$

32. Prepare an alignment chart for the equation

$$D^3 = D_H^3(1 - n^4)$$

where

D = diameter of solid shaft in inches
D_H = outside diameter of equivalent hollow shaft in inches
$n = \frac{d_H}{D_H}$ (d_H = inside diameter hollow shaft)
D to vary from 1.5 to 10
D_H to vary from 1.5 to 12.

33. Solve Problem 32, substituting for the limits of D_H, limits of n = 0.4 to 0.9.

34. Prepare a three parallel scale alignment chart for the equation

$$t = \frac{\sqrt{0.2n}}{S}$$

n to vary from 12 to 80
S to vary from 1 to 10.

35. Given the equation

$$Q = 15.88\, d^2 \sqrt{H}$$
d to vary from $\frac{1}{2}$ to 4
H to vary from 1 to 60.

Make a three parallel scale alignment chart, having no scale less than 8 inches or more than 10 inches long.

Show extreme values, and at least two intermediate values on each scale. Show an index line in position to solve $d = 2, H = 15, Q = ?$

36. Construct a chart representing the equation

$$H^3 = \frac{W^2}{11B^2}$$

H to vary from 0.2 to 2
B to vary from 0.5 to 50.

Scales to be not less than 8 inches long.
Show an index line in position to solve

$$H = 1.5, \quad B = 30, \quad W = ?$$

37. Prepare a three parallel scale alignment chart for the equation

$$v = 0.0796d^{0.63}h^{0.54}$$

h to vary from 1 to 100
d to vary from 3 to 60.

38. Make a three parallel scale alignment chart for the equation

$$\sqrt{d} = 1.75\sqrt[3]{\frac{T}{f}}$$

f to vary from 2000 to 25,000
T to vary from 20,000 to 150,000.

Scales to be >8 inches but < 11 inches long.
Show an index line in position to solve

$$T = 100{,}000, \quad f = 10{,}000, \quad d = ?$$

39. Prepare an alignment chart for the equation

$$A = R^2\left[\frac{\pi}{180}\cos^{-1}(1-k) - (1-k)\sqrt{2k-k^2}\right]$$

A to vary from 0.4 to 100
R to vary from 1 to 10.

40. Prepare a three parallel scale alignment chart for the equation

$$P = CD^{\frac{1}{3}}F^{\frac{1}{2}}$$

D to vary from 0.1 to 1
F to vary from 0.01 to 0.2.

Graduate the P scale on one side for $C = 45{,}000$, and on the other for $C = 70{,}000$.

41. Make a three parallel scale chart for the equation

$$T = \frac{N}{\cos^3\alpha}$$

N to vary from 10 to 100
α to vary from 10° to 70°.

(*Suggestion.* — Let $c = \cos \alpha$ and proceed to lay down scales T, N, and c. The c scale may then be graduated in terms of α in accordance with tabular values, thus,

α°	$c = \cos \alpha$
10	.985
20	.940
etc.	
70	.342)

CHAPTER III

EQUATIONS OF MORE THAN THREE VARIABLES (PARALLEL SCALES AND Z CHART)

PARALLEL scale charts may be prepared for equations involving any number of variables provided the equation can be put in the form

$$x + y + w + u + \text{etc.} = z$$

where each term contains only one variable. As has been previously shown a variable may appear in any complex form provided its use is confined to one term.

Charts for equations of the above form use one additional ungraduated scale for each variable more than three. These ungraduated scales are known as pivot or turning scales. The principles underlying the construction of charts for three variables apply with slight amplification to equations of similar form containing more than three variables.

For an equation of four variables of the form

$$x + y + w = z$$

substitute two equations of three variables, as

$$x + y = Q$$
$$Q + w = z.$$

Two charts might then be prepared each having three parallel scales. Given values of any three of the variables in the original equation the fourth may be found by obtaining a value of Q from one of the charts and substituting in the other. This necessitates graduating two Q scales. This may be avoided by combining the two charts, thereby using only one Q scale which need carry no graduations, the index lines of the two charts

then meeting in a common point on the Q scale, whose value need not be known. This Q scale is the pivot or turning scale and there must be one turning scale for each variable more than three. It should be noted that Q must have the same sign in both equations or the graduations would be reversed. Judgment in selecting the two variables whose sum is to equal Q, and the position of Q in the second equation, viz., in the X scale position or Y scale position has much to do with the preparation of a satisfactory chart. No general rules can be given for the best arrangement of scales.

An illustrative example will be worked out.

Example 4. Prepare a parallel scale alignment chart for the equation

$$P = 0.196\frac{d^3f}{r}$$

limits to be as follows:

d to vary from 0.10 to 0.50
f to vary from 30,000 to 100,000
r to vary from 0.5 to 2.0.

This equation having four variables will require five scales, one of which will be an ungraduated turning scale. Since no limits are imposed upon P it is desirable that it be made the Z scale of the second chart. The following combinations are therefore possible, the constant in each case appearing as a coefficient of P.

(1) $d^3f = Q$ and $Q = 5.1\,Pr$

$$3\log d + \log f = \log Q$$

and

$$\log Q - \log r = \log P + K$$

(2) $\dfrac{d^3}{r} = Q$ and $Q = \dfrac{5.1P}{f}$

$$3\log d - \log r = \log Q$$

and

$$\log Q + \log f = \log P + K$$

$$(3) \qquad \frac{f}{r} = Q \quad \text{and} \quad Q = \frac{5.1P}{d^3}$$

$$\log f - \log r = \log Q$$

and $$\log Q + 3 \log d = \log P + K.$$

The constant, K, equals log 5.1 in each case.

Assuming Case (1) the arrangement of scales and moduli values are determined as follows:

$$3 \log d + \log f = \log Q$$

in form $$x + y = z.$$

Since d varies from 0.10 to 0.50

$\log d$ varies from -1 to -0.3

$x = 3 \log d$ varies from -3 to -0.9

or a range for x of 2.1 units.

For a scale 6 ins. long

$$M_x = \frac{6}{2.1} = 2.88.$$

Assume $M_x = 3$, $M_{\log d} = 9$, scale length 6.3 ins.

Since f varies from 30,000 to 100,000, $y = \log f$ varies from 4.48 to 5 or a range of 0.52 units for y.

Assuming a 6 ins. scale length

$$M_y = \frac{6}{0.52} = 11.5.$$

For convenience in laying out scales make $M_y = 10$, $M_{\log f} = 10$ and scale length 5.2 ins.

The value of $M_{\log Q} = M_z$ may now be obtained by the use of equation (2) (page 24).

$$M_{\log Q} = M_z = \frac{M_x M_y}{M_x + M_y} = \frac{3 \times 10}{3 + 10} = 2.31.$$

The second equation for this case is

$$\log Q - \log r = \log P + K$$

the constant K being the logarithm of the coefficient of P, viz., 5.1 whose value need not be determined as will appear later.

The form of this second equation is

$$x - y = z + K$$

the X scale of this equation being the Z scale of the first equation, viz., the Q scale. The modulus of the Q scale ($M_{\log Q} = 2.31$) having been found previously, now becomes the modulus of the X scale, or

$$M_x = 2.31$$

Since r varies from 0.5 to 2.0

$y = \log r$ varies from -0.3 to 0.3, or range of 0.6.

Assume

$$M_y = M_{\log r} = 10,$$

giving a scale length of 6 inches.

It should be noted that the minus sign for $\log r$ will merely *invert* this scale.

The scale modulus for the Z or P scale is now found, using equation (2) (page 24).

$$M_z = M_{\log P} = \frac{2.31 \times 10}{2.31 + 10} = 1.87.$$

All scale moduli being known the arrangement and spacing of the scales may be shown by two sketches, one for each equation, (Fig. 19). The spacing of the scales for the first equation is shown in Fig. 19 (*a*) and is obtained by the use of equation (1) (page 24), giving

$$\frac{r}{s} = \frac{M_x}{M_y} = \frac{3}{10}.$$

For the second equation the scales are spaced as shown in Fig. 19 (*b*), found from the relation

$$\frac{r}{s} = \frac{M_x}{M_y} = \frac{2.31}{10}.$$

The units for sketch, Fig. 19 (*a*), are not necessarily the same as the units for sketch, Fig. 19 (*b*), thus permitting the placing of the *f* scale and the *r* scale in coincidence when the two charts are combined with a common *Q* scale.

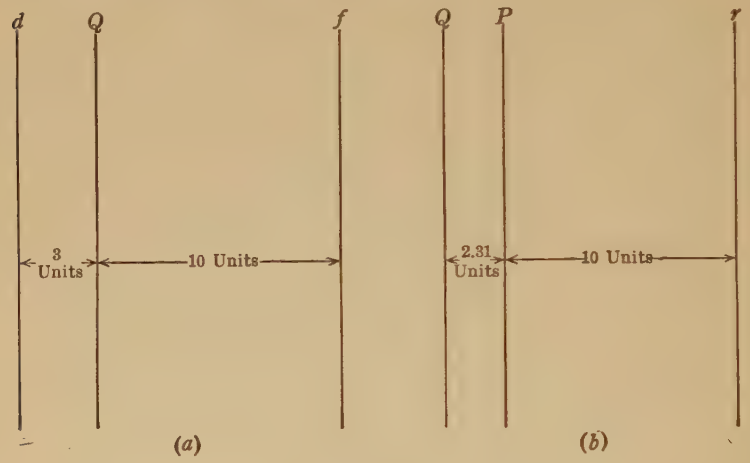


FIG. 19.

The combination of the two charts is shown in Fig. 19 (*c*). The scales having been properly located, the *d*, *r*, and *f* scales may be graduated by folding on the logarithmic scales as follows:

d scale, fold on log scale 9 ins.
f scale, fold on log scale 10 ins.
r scale, fold on log scale 10 ins. and invert.

A starting point for the *P* scale is found by assuming convenient values for *d*, *f*, and *r* within the given ranges, and solving the original equation for *P*, as

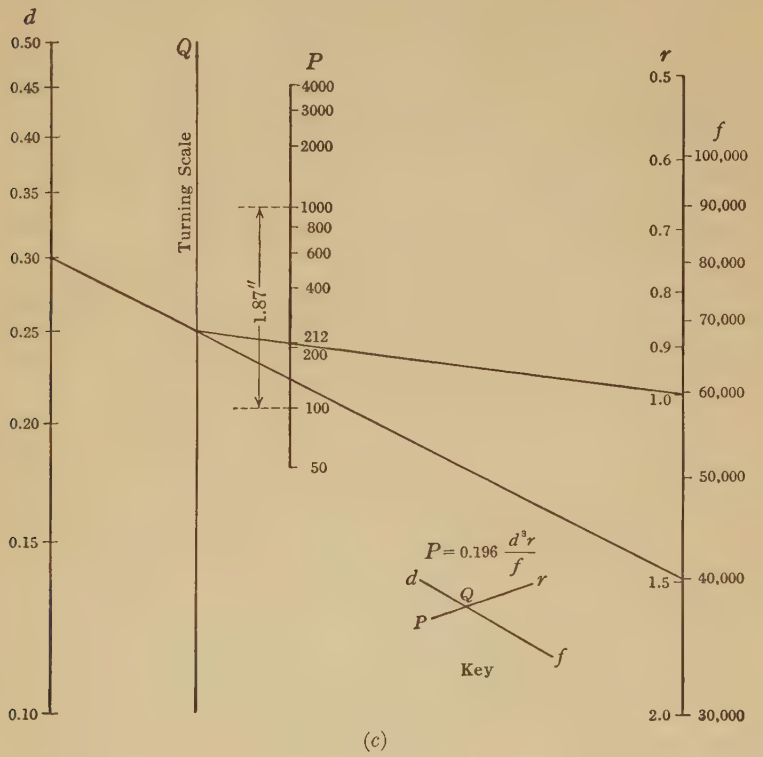
$$d = 0.3, \quad f = 40{,}000, \quad r = 1,$$

giving

$$P = 0.196\,\frac{(0.3)^3 40{,}000}{1}$$

$$= 212.$$

Hence, drawing index line $d = 0.3, f = 40{,}000$ determines a value on the Q scale (not known) which when connected with the value 1 of the r scale by an index line must cross the P scale at the value 212. The P scale may now be graduated by folding on log scale 1.87 ins.

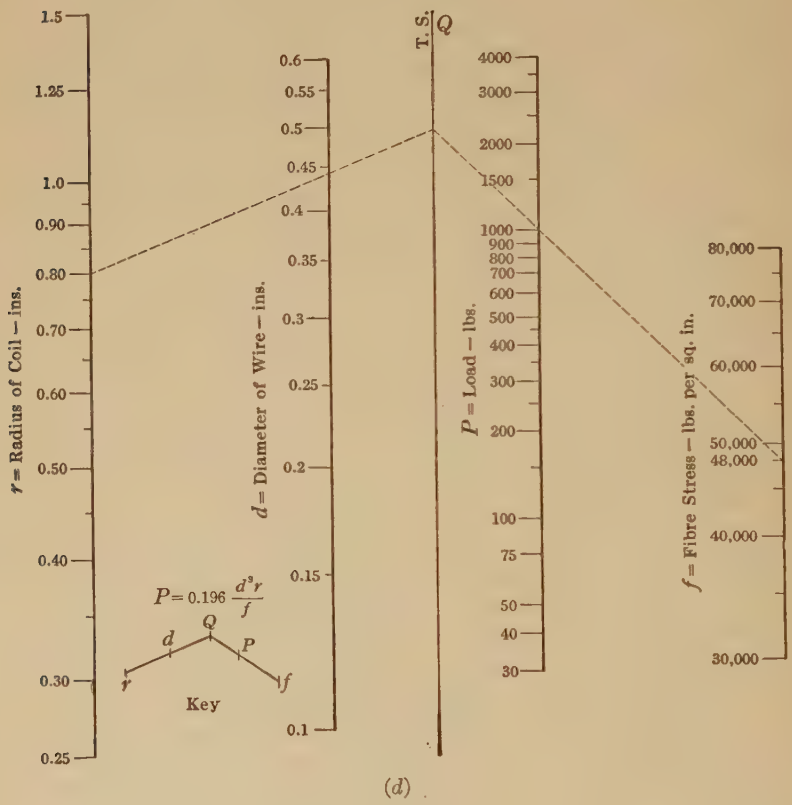


(c)

FIG. 19.

Figure 19 (*d*) shows the chart resulting from the assumptions of equations in Case (2), with slight modifications. In this solution the Q scale becomes the Y scale of the first equation and the X scale of the second equation. In imposing limits on the variable shown on the Z scale, as is done in the first case, and

thereby determining M_z care should be taken to make this modulus less than M_x. Otherwise, no solution is possible. (See page 42.)



(*d*)

FIG. 19.

Many solutions are possible and it may be necessary to make several trials before a satisfactory solution is obtained.

A chart for a four variable equation in which the turning scale is made the Z scale for both equations gives a very satis-

factory solution. As an example take the equation

$$t = \frac{pr}{f}$$

or $$tf = pr.$$

Letting $$pr = Q$$

it follows that $$ft = Q.$$

Hence $$\log p + \log r = \log Q$$

and $$\log f + \log t = \log Q.$$

FIG. 20.

Figures 20 (*a*) and 20 (*b*) show the relative positions of the scales for the equations.

Superimposing these figures gives a final chart as shown in Fig. 21 with the turning scale as the middle scale. A difficulty may

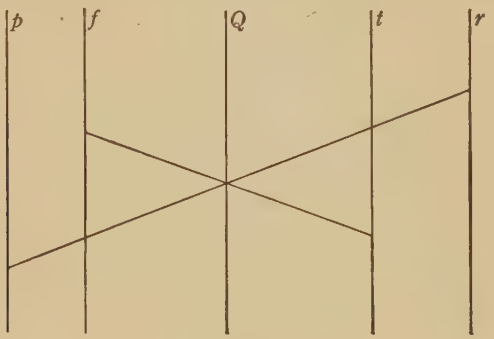


FIG. 21.

arise in this case, however, when one of the scale moduli becomes a negative quantity. Imposing limits upon the variables p, r,

and f, the scale modulus for Q may be found from the first equation. This value then becomes the scale modulus for the Z scale of the second equation and it is necessary to solve for the modulus of the Y scale, viz., $M_y = M_{\log t}$ of the second equation, since no limits were imposed upon the variable t. No solution is possible, unless M_z is less than either M_x or M_y. This may be shown as follows:

It is known that

$$M_z = \frac{M_x M_y}{M_x + M_y}.$$

Let $$K = \frac{M_z}{M_x}.$$

Then $$KM_x = \frac{M_x M_y}{M_x + M_y},$$

and $$M_y = M_x\left(\frac{K}{1 - K}\right).$$

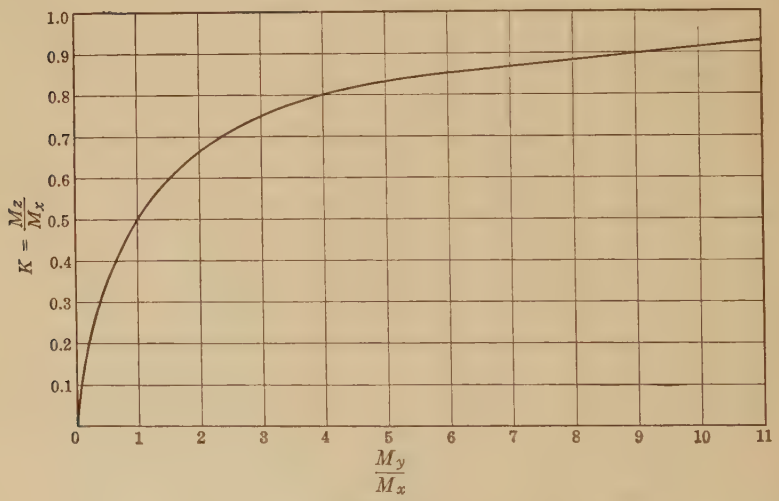


FIG. 22.

If M_y and M_x are positive, K must be less than unity, viz., M_z must be less than M_x. Similarly it may be shown that M_z is less than M_y.

The relation of the ratio $\frac{M_y}{M_x}$ with respect to $\frac{M_z}{M_x}$ is shown in Fig. 22. For satisfactory charts the latter ratio should, in general, be not less than 0.3 or more than 0.8.

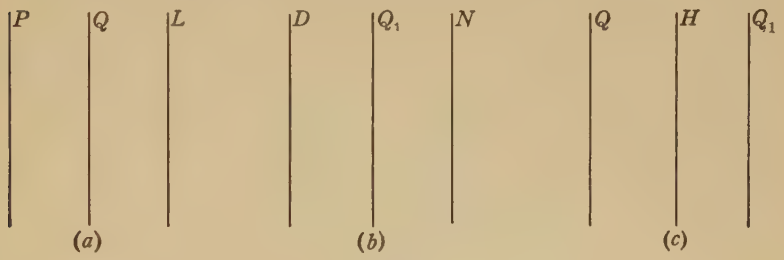


FIG. 23.

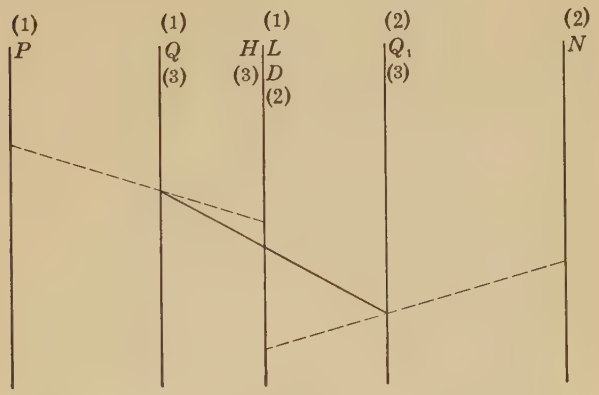


FIG. 24.

A familiar equation having five variables is that for the horse power developed by a steam engine, viz.,

$$H = \frac{PLAN}{33{,}000}$$

or $H = 0.000001983\ PLD^2N$. (All linear dimensions in inches.) An alignment chart for this equation will require two turning

scales. The equation may be split into three equations of three variables each, thus

$$PL = Q$$
$$D^2N = Q_1$$
$$QQ_1 = 504{,}000\,H$$

limits being imposed upon all variables except H.

The positions of the scales for the three equations are shown in Fig. 23 (a, b, c). Combining the three figures into a single chart gives the solution shown in Fig. 24, where, for simplicity, the L and D scales have been combined. Index lines connect scales similarly numbered.

Special Equations having Parenthetical Constants. Charts may often be prepared for equations having a constant in a parenthesis. This usually requires a shift of points on the turning scale. An example of this type will be given.

Example 5. Prepare an alignment chart for the equation

$$W = w\left(\frac{N^2h}{35{,}200} - 1\right).$$

The limits of the variables to be

N, 50 to 500
h, 2 to 8
w, 1 to 4

The equation having four variables may be written as two equations of three variables, thus

$$N^2h = 35{,}200\,Q$$
$$\frac{W}{w} = Q - 1 = Q_1.$$

The scale arrangements for these equations is shown in Fig. 25 (a, b).

It is evident that, if four variables are to satisfy the original

equation, the index line connecting values of N and h in Fig. 25 (a) must cut the scale Q in a value one unit greater than that shown by an index line from W to w (Fig. 25 (b)) on scale Q_1. If the Q and Q_1 scales are superimposed it will be necessary to allow for this difference of values on the Z scales, viz., the turn-

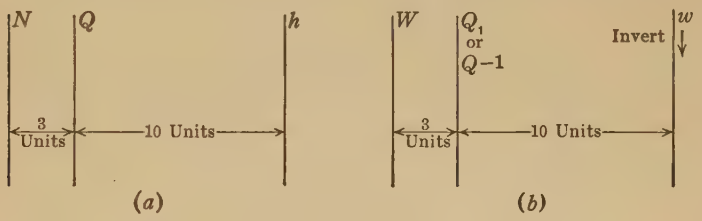


FIG. 25.

ing scale of the chart. To enable the turning to be made on the same value (not the same point) of the turning scales the graduations of one of the scales may be arbitrarily changed, as will be clear from the following solution.

Taking logarithms of the preceding equations

$$2 \log N + \log h = \log Q + K$$

and

$$\log W - \log w = \log Q_1.$$

In the first equation the limits of N and h give

$$M_x = 3,\ M_{\log N} = 6$$
$$M_y = 10,\ M_{\log h} = 10$$

hence

$$M_z = 2.31,\ M_{\log Q} = 2.31.$$

For simplicity the modulus of the Z scale in the second equation is made the same as that for the first equation, these being the turning scales.

Therefore, $M_z = 2.31$ and $M_{\log Q_1} = 2.31$.

From the limits for the variable w, the modulus M_y for the second equation is found to be

$$M_y = 10,\ M_{\log w} = 10.$$

Hence from equation (2) (page 24)

$$2.31 = \frac{10\,M_x}{10 + M_x}$$

and

$$M_x = 3,\ M_{\log W} = 3.$$

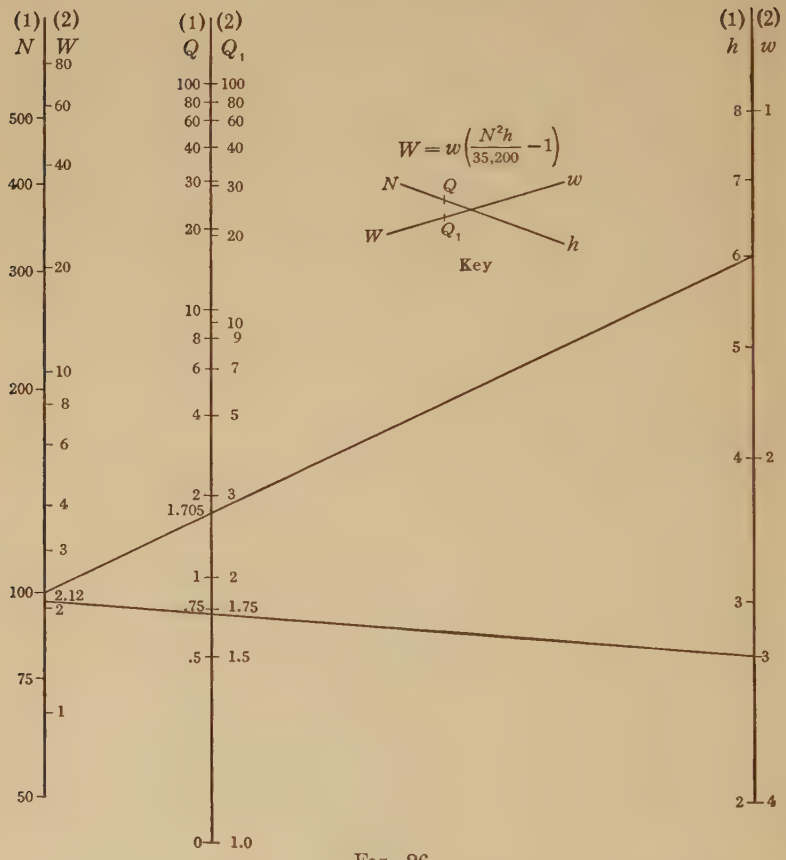


Fig. 26.

Figures 25 (*a*) and 25 (*b*) show the relative spacings of the several scales, it being understood that the units of the figures are not necessarily the same, although for simplicity it is desirable to do so in this case.

Figure 26 shows the chart with the turning scales superimposed. The fact that the other scales coincide is peculiar to this solution. Values of the variables which satisfy the equation may be found by drawing index lines, one of which connects values of the variables on the left side of the scales, viz., N, h, and Q, and the other the scales of W, w, and Q_1. Since the value of Q must be one unit more than that of Q_1 ($Q_1 = Q - 1$), the respective index lines must cut the turning scale at values of Q and Q_1 differing by unity. For convenience the values of the Q_1 scale have been increased by unity, viz. a value of $Q_1 = 2$ is marked 3, a value of 19 is marked 20, etc. With this graduation the index lines may be said to turn on the same value (not the same point) of the turning scale. For values near the upper end of the turning scale the effect of the constant is negligible. The index lines shown determined the starting point for the W scale, as follows:

Assuming, $N = 100, \quad h = 6, \quad w = 3$

the original equation gives,

$$Q = 1.705, \quad Q_1 = 0.705 \quad \text{and} \quad W = 2.12.$$

The scales may be graduated by folding on the proper log scales.

THE Z OR N CHART

A Z or N chart may be prepared for equations of three or four variables which can be written in the form

$$\frac{x}{y} = \frac{z}{w}.$$

In the case of three variables one of the variables becomes a constant.

Figure 27 shows the positions of the four scales placed on two parallel lines. If values of the variables are to satisfy the equation, an index line connecting values of x and y must intersect an index line connecting values of z and w on the diagonal OO_1.

This chart may also be placed with its scales horizontally (Z Chart), or the scales may be placed as shown in Fig. 28 with a

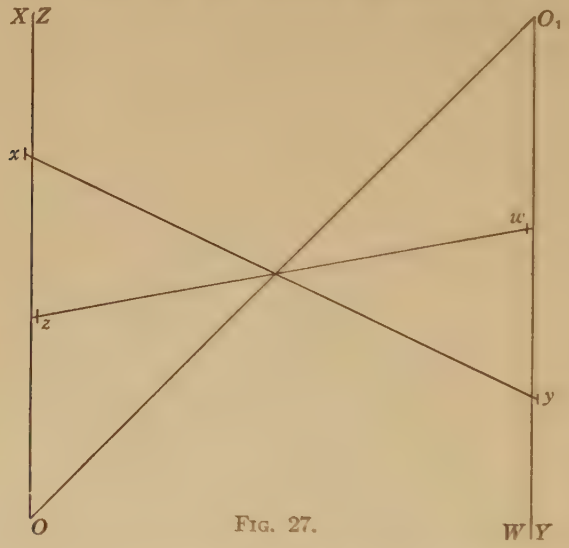


Fig. 27.

low range for the variables shown on the inside of the scales and a high range on the outside.

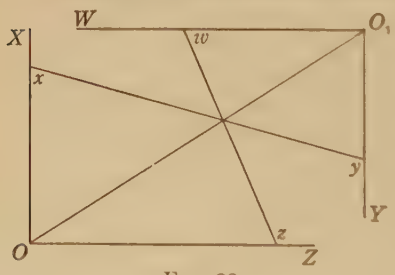


Fig. 28.

All scales must start with the zero values at points O and O_1. Negative values of the variables may be shown by continuing the scales beyond the points O and O_1. This is necessary when showing values less than unity of a variable appearing in the logarithmic form.

Determination of Relation of Scale Moduli for Z Charts. Given equation of the form

$$\frac{x}{y} = \frac{z}{w}.$$

It is desired to obtain the relation between M_x, M_y, M_z and M_w, to enable the determination of the fourth when any three have been arbitrarily chosen.

Let a, b, c and d be specific values for the variables such that

$$\frac{a}{b} = \frac{c}{d}.$$

Referring to Fig. 29, the distance from the zero point O to a point $x = a$ on the X scale is aM_x. Similarly for the points

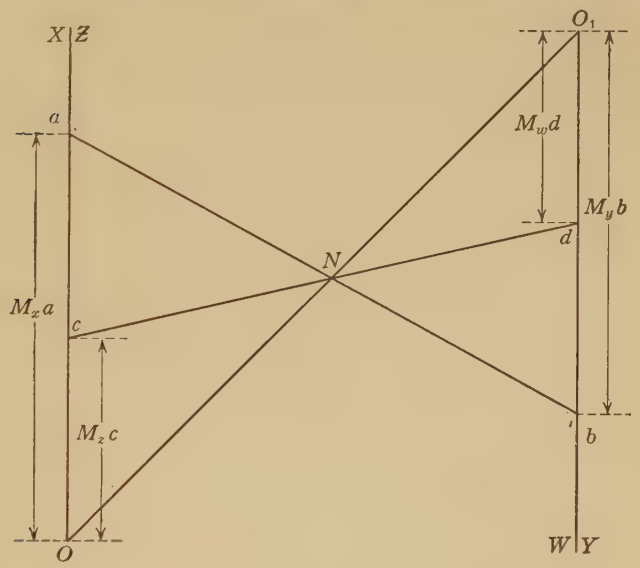


FIG. 29.

b, c and d, the distances are bM_y, cM_z and dM_w.

From similar triangles OcN and NO_1d

$$\frac{cM_z}{dM_w} = \frac{ON}{O_1N}$$

and from triangles OaN and NO_1b

$$\frac{aM_x}{bM_y} = \frac{ON}{O_1N}.$$

Therefore, $$\frac{aM_x}{bM_y} = \frac{cM_z}{dM_w}$$

but since $$\frac{a}{b} = \frac{c}{d}$$

$$\frac{M_x}{M_y} = \frac{M_z}{M_w}$$

or $$M_w = \frac{M_y M_z}{M_x} \qquad (3)$$

an equation expressing the necessary relation of the moduli.

An example will illustrate the use of this chart.

Example 6. Prepare a Z chart for the solution of the equation

$$b = \frac{4.56\ pP}{f}$$

p to vary from 0 to 10

P to vary from 0 to 5,000

f to vary from 0 to 20,000.

Rewrite the equation placing the coefficient and the variable for which limits are not given in the denominator of the second ratio, as

$$\frac{f}{p} = \frac{P}{0.22b}$$

which is in the form $\frac{x}{y} = \frac{z}{w}$.

The limits for f and therefore for x $(x = f)$, show a range of 20,000. Assuming a scale 5 ins. long

$$M_x = M_f = \tfrac{5}{20000} = \tfrac{1}{4000} = 0.00025.$$

Similarly for p,

$$M_y = M_p = \tfrac{5}{10} = \tfrac{1}{2} = 0.5,$$

and for P,

$$M_z = M_P = \tfrac{5}{5000} = \tfrac{1}{1000} = 0.001$$

Therefore from equation (3)

$$M_w = \frac{\frac{1}{2} \times \frac{1}{1000}}{\frac{1}{4000}} = 2$$

and $$M_b = 0.22\, M_w \text{ (since } w = 0.22\, b)$$
$$= 0.44.$$

The completed chart is shown in Fig. 30. The scales are uniform and, with the exception of the "b" scale, may conven-

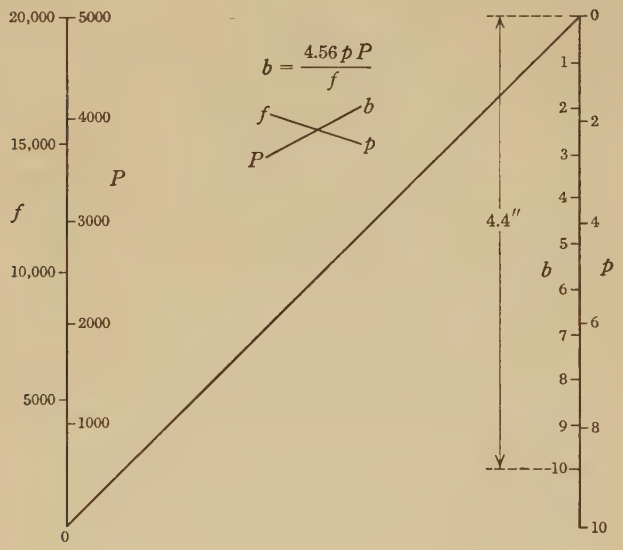


Fig. 30.

iently be graduated by means of an engineer's scale. The "b" scale may be marked off by a graduated uniform scale on which a length of 4.4 inches is divided uniformly into ten divisions and sub-divisions.

The following example will serve to illustrate the method of preparing a chart for an equation in which the variables do not appear to the first power, thus requiring non-uniform scales.

Example 7. Prepare a Z chart for the graphical solution of the equation

$$S = \frac{NP^{\frac{1}{2}}}{H^{\frac{3}{4}}}.$$

The following limits are imposed

N, 0 to 1000
S, 0 to 50
H, 0 to 200

Rewrite the equation in the form

$$\frac{N}{S} = \frac{H^{\frac{3}{4}}}{P^{\frac{1}{2}}}$$

The equation is now in the form $\frac{x}{y} = \frac{z}{w}$.

Assuming scale lengths of 5 ins. (approximately), the scale moduli for N and S may be obtained from the range of limits given, as follows:

$$M_x = M_N = \tfrac{5}{1000} = \tfrac{1}{200} = 0.005$$
$$M_y = M_S = \tfrac{5}{50} = \tfrac{1}{10} = 0.1.$$

Since the variable H has an exponent other than one, and $z = H^{\frac{3}{4}}$, the range for z and H will not be the same. To determine M_z, note that

H varies from 0 to 200

and $z = H^{\frac{3}{4}}$ varies from 0 to 752

therefore, for a scale length of 5 inches,

$$M_z = M_{H^{\frac{3}{4}}}$$
$$= \tfrac{5}{752} \quad \text{or} \quad \tfrac{1}{150} = 0.0067 \text{ (nearly)}.$$

This change in M_z will not noticeably affect the scale length. The scale modulus M_w may now be obtained

$$M_w = \frac{M_y M_z}{M_x}$$

or

$$M_w = M_{P^{\frac{1}{2}}} = \frac{\frac{1}{10} \times \frac{1}{150}}{\frac{1}{200}} = \tfrac{2}{15} = 0.134.$$

Scales N and S may be readily laid down by the use of uniform scales with the moduli found.

To graduate the H scale note that while the scale must show values of H, the scale modulus (M_z) is for values of $H^{\frac{3}{4}}$, and the following method may be used to find as many points as may be desirable. Only four points will be found here.

Tabulate values of H and corresponding values of $H^{\frac{3}{4}}$.

H	50	100	150	200
$H^{\frac{3}{4}}$	133	316	525	752

Since the scale modulus $M_z = M_{H^{\frac{3}{4}}}$ is to be 0.0067, a uniform scale 6.7 inches long may be used to show values of z, viz., $H^{\frac{3}{4}}$, from 0 to 1000. Use this scale therefore, marking opposite points 133, 316, 525 and 752, values of H = 50, 100, 150 and 200 respectively.

A similar method of procedure may be used for the P scale. Again, only four points are tabulated, viz., P = 100, 400, 1000, 1600.

P	100	400	1000	1600
$P^{\frac{1}{2}}$	10	20	31.6	40

The scale modulus $M_w = M_{P^{\frac{1}{2}}} = 0.134$, therefore a uniform scale 1.34 inches long may be used to show an increase of w or $P^{\frac{1}{2}}$ of 10 units. Using such a scale the values of P selected may be marked at 10, 20, 31.6 and 40 units. It is evident that the same result might be obtained by using an engineer's scale and marking points P at the following distances.

P = 100 at $10 \times 0.134 = 1.34$ ins.
P = 400 at $20 \times 0.134 = 2.68$ ins.
P = 1000 at $31.6 \times 0.134 = 4.23$ ins.
P = 1600 at $40 \times 0.134 = 5.36$ ins.

Figure 31 shows the completed chart.

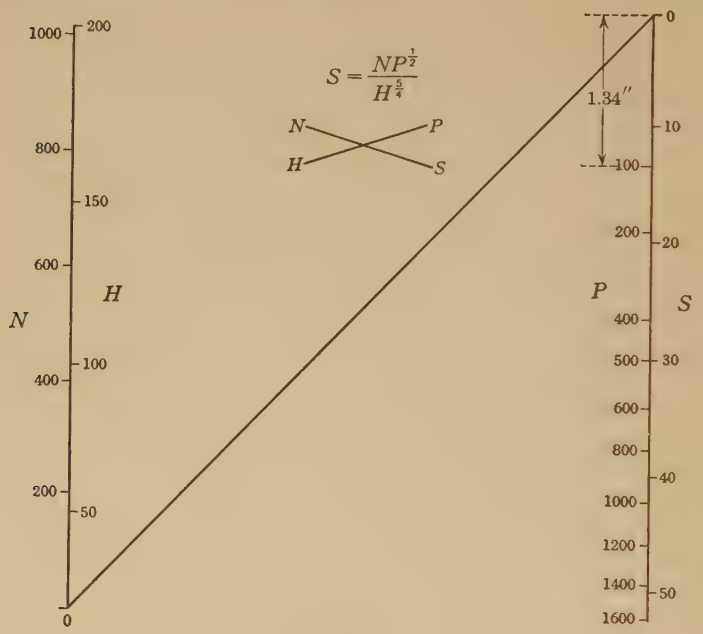


FIG. 31.

Z Charts for Three Variables. As was stated earlier a Z chart may be prepared for an equation of three variables which can be put in the form

$$\frac{K}{y} = \frac{z}{w}$$

where K is a constant, which may occupy the position of any one of the variables. However, since the scale carrying the constant will have only one graduated point, that of the constant, it is well to assume the modulus for this scale such that the point will lie within the limits of the diagram. This necessitates assigning limits for only two of the variables.

The following example will illustrate such a case.

Example 8. Prepare a Z chart for the equation

$$q = 3.33\, BH^{\frac{3}{2}}$$

with the following limits

B to vary from 0 to 10

H to vary from 0 to 3.

Rewrite the equation in the form of the equality of two ratios, as

$$\frac{0.3}{B} = \frac{H^{\frac{3}{2}}}{q}.$$

The chart for such an equation differs from that having the four variables only in the fact that the index line is always drawn from the same point on the X scale, viz., $x = 0.3$.

Assuming scale lengths of approximately 5 inches, the scale moduli are determined.

Take $M_x = 15$, giving a single point K (Fig. 32) on the X scale $0.3 \times 15 = 4.5$ inches from the zero end of the scale.

Modulus $M_y = M_B = 0.5$ from the limits given.

To find M_z, note that for $H = 3$

$$H^{\frac{3}{2}} = 5.2,$$

hence, since $z = H^{\frac{3}{2}}$

$$M_z = \frac{5}{5.2} \quad \text{or} \quad 1 \text{ (nearly)},$$

which is also $M_{H^{\frac{3}{2}}}$, giving a scale length of 5.2 inches.

Scale modulus for q may now be found

$$M_w = M_q = \frac{0.5 \times 1}{15} = \frac{1}{30} = 0.033.$$

The B scale is readily graduated along the Y scale, shown as B_1 in Fig. 32.

To graduate the H scale, find corresponding values of H and $H^{\frac{3}{2}}$, as

H	0	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3
$H^{\frac{3}{2}}$	0	1	1.84	2.83	3.91	5.2

and using the scale modulus $M_{H^{\frac{3}{2}}} = 1$ mark off points as given for $H^{\frac{3}{2}}$ and graduate them as the corresponding value of H.

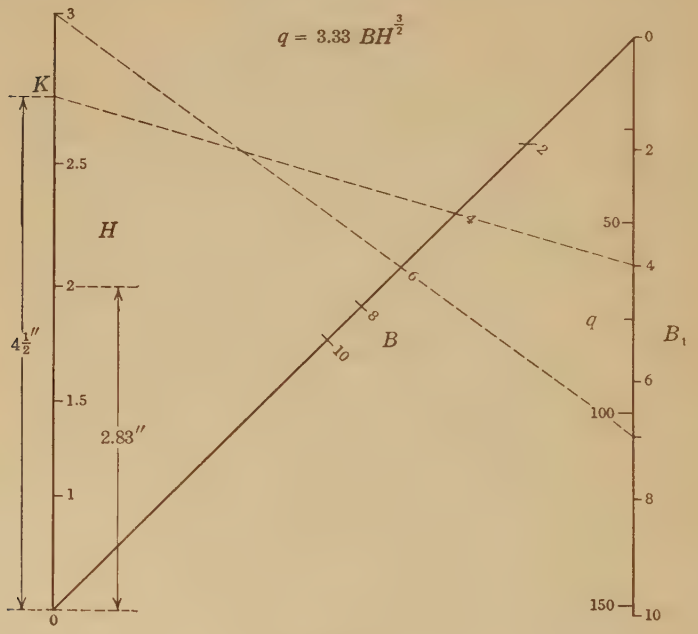


FIG. 32.

The W or q scale may be graduated as a uniform scale by using a scale 3.3 inches long for an increase of 100 units in the value of q.

Since all index lines from K to B_1 must start at the point K, this chart may be simplified by showing the graduations for B

on the diagonal line. Values of H, B, and q which satisfy the equation will now fall on a single straight line. The index line drawn dotted shows $q = 104$, for $H = 3$ and $B = 6$.

Equations with Variable Exponent. Equations of the form

$$p = v^n$$

where p, v, and n are variable may be charted on a three variable Z chart.

Taking logarithms the equation becomes

$$\log p = n \log v,$$

this may be written

$$\frac{\log p}{\log v} = \frac{n}{1},$$

which is in the form

$$\frac{x}{y} = \frac{z}{w}$$

with $w = 1$.

The diagonal from $x = 0$ to $y = 0$, will in this case run from $p = 1$ to $v = 1$ (Fig. 33), since $x = \log p$ and $y = \log v$. Values of p greater than unity will be shown above the point $p = 1$, while values of v greater than unity will be shown below $v = 1$, as is the case in the Z chart.

To graduate the diagonal in terms of n proceed as follows:—

Note that the scale moduli for x, y and z are

$$M_x = M_{\log p}$$
$$M_y = M_{\log v}$$

and

$$M_z = M_n.$$

Hence, substituting in equation (3) (page 50), obtain

$$M_w = \frac{M_{\log v} M_n}{M_{\log p}}.$$

Since the value of w is always unity this equation gives the distance to the constant point on the w scale.

Values of n on the Z scale may be transferred to the diagonal $p = 1$, $v = 1$, by drawing lines from the constant point $w = 1$ to the values of n and marking the intersection with the diagonal

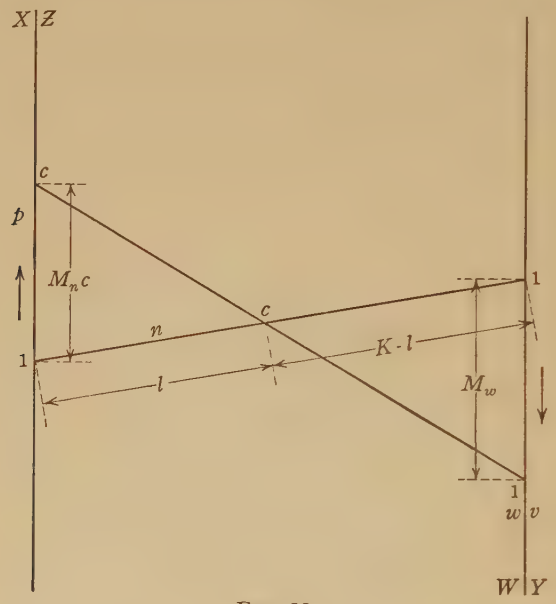


FIG. 33.

correspondingly. As shown in Fig. 33 the value $n = c$ is thus transferred.

To graduate the diagonal directly in terms of n assume K as the length of the diagonal and note that in the similar triangles shown (Fig. 33)

$$\frac{l}{M_n c} = \frac{K - l}{M_w},$$

or, substituting the value for M_w,

$$\frac{l}{M_n c} = \frac{(K - l) M_{\log p}}{M_{\log v} M_n},$$

hence
$$lM_{\log v} = cKM_{\log p} - clM_{\log p}$$
$$l(M_{\log v} + cM_{\log p}) = cKM_{\log p}$$
$$l = \frac{cKM_{\log p}}{M_{\log v} + cM_{\log p}}.$$

From this equation the distances from the point $p = 1$ to points $n = c$ on the diagonal can be determined directly.

For the special case of $M_{\log p} = M_{\log v}$ the equation becomes

$$l = \left(\frac{c}{c+1}\right)K.$$

An example will illustrate the method of constructing a chart for an equation with a variable exponent.

Example 9. Construct an alignment chart for the solution of the equation

$$T = V^{n-1}.$$

T to vary from 0.1 to 10

V to vary from 0.1 to 100.

Using logarithms the equation may be written in the form

$$\frac{x}{y} = \frac{z}{w},$$

thus,
$$\frac{\log T}{\log V} = \frac{n^1}{1}$$

where
$$n^1 = n - 1.$$

From the limits for T the scale modulus for x and its equal $\log T$ (range 2 units) is found to be (assuming scales 6 inches long)

$$M_x = M_{\log T} = 3$$

and similarly from the limits for V, the scale moduli M_y and $M_{\log V}$ are found to be

$$M_y = M_{\log V} = 2.$$

The distance from point $T = 1$ along the diagonal to point $n^1 = 1$, $(n = 2)$ is (assuming $K = 5$)

$$l = \frac{1 \times 5 \times 3}{2 + 1 \times 3}$$
$$= \tfrac{15}{5} = 3''.$$

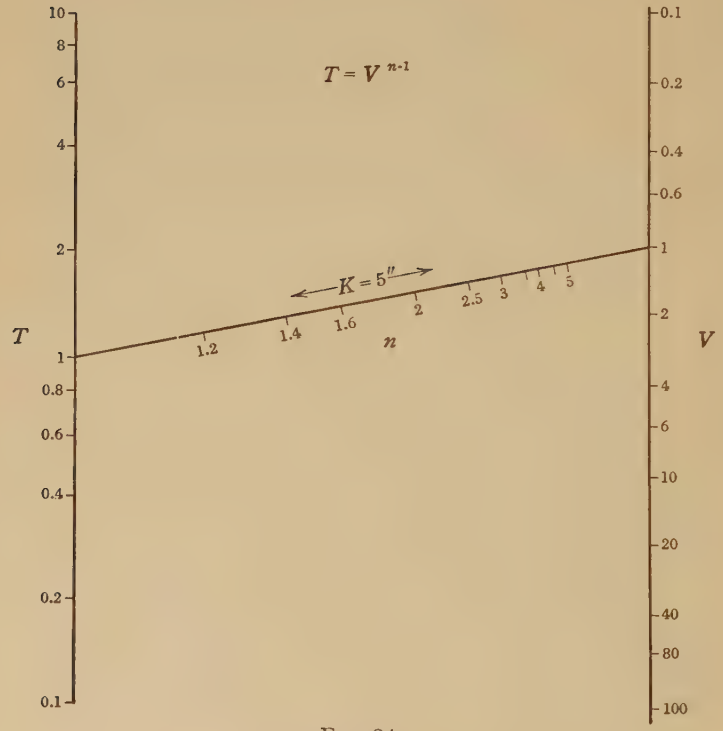


Fig. 34.

Other values of n^1 (and therefore n) may be obtained in like manner. The following table shows values of l for the charted values of n.

The completed chart is shown in Fig. 34. The V scale is inverted as is the case for the Y scale in a Z chart.

n	n^1	l
1	0	0
1.2	0.2	1.15
1.4	0.4	1.88
1.6	0.6	2.37
2.00	1.00	3.00
2.50	1.50	3.46
3.00	2.00	3.75
3.50	2.50	3.95
4.00	3.00	4.09
4.50	3.50	4.20

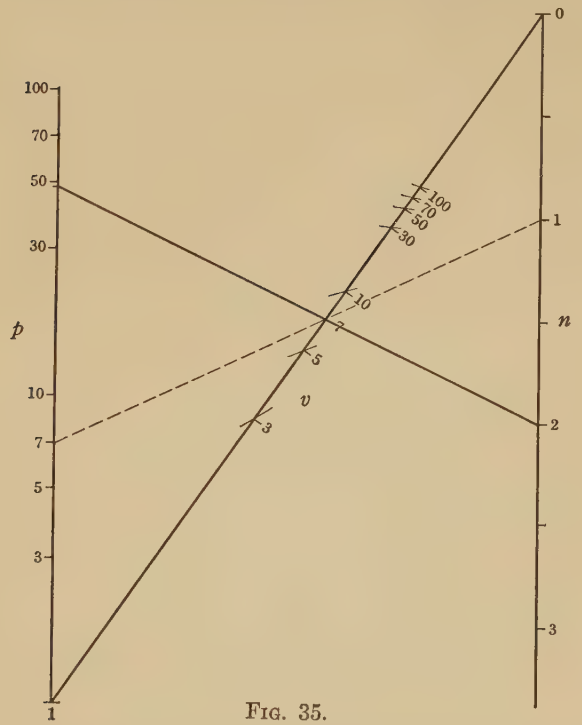


FIG. 35.

Figure 35 shows a chart for determining powers and roots based on the equation

$$p = v^n,$$

or

$$\sqrt[n]{p} = v.$$

For this chart, write

$$\frac{\log p}{n} = \frac{\log v}{1}$$

which is in the form $\frac{x}{y} = \frac{z}{w}$ with unity as the value of w.

If the same scale modulus is assumed for the Y and W scales that is $M_y = M_w$, it follows that $M_n = M_w$.

From equation 3 (page 50) it is found that

$$M_x = M_z$$

or

$$M_{\log p} = M_{\log v}.$$

With scale moduli so assumed, the p and v scales coincide, as do the n and w scales.

Hence the v scale may be graduated on the diagonal by drawing lines from the point $n = 1$ (that is $w = 1$, since $M_n = M_w$) to points on the p scale and marking the value of v equal to p on the diagonal as shown by the line $p = 7$, $n = 1$ (Fig. 35). Any index line will cut the three scales in values such that p is the nth power of v, or v is the nth root of p. The full index line drawn in Fig. 35, shows $7^2 = 49$.

Equations of the form just discussed may be put in the general form $x + y = z$ and a three parallel scale chart constructed. To do this take logarithms twice, thus

$$p = v^n$$

$$\log p = n \log v$$

$$\log \log p - \log \log v = \log n$$

which equation is in the form $x - y = z$. However, log log scales are not, in general, satisfactory since no values of the variables p and v unless greater than unity can be included on such a chart, and the scale moduli are large for medium ranges of the variables.

Z Charts for More Than Four Variables. Equations which may be put in the form

$$x = \frac{yzw^1}{y^1z^1}$$

may be charted using a double Z chart having one turning scale.

Assuming $$Q = \frac{y^1z^1}{w^1}$$

rewrite the equation as $$\frac{x}{y} = \frac{z}{Q}$$

and $$\frac{Q}{y^1} = \frac{z^1}{w^1}.$$

Charts for the above equations may be prepared under the principles already laid down and the scales arranged as in Fig. 36.

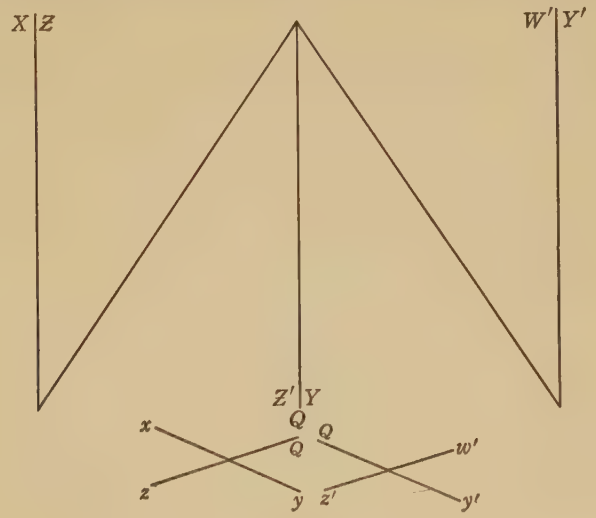


FIG. 36.

Charts for equations of the type $\frac{x}{y} = \frac{z}{w}$ may be prepared using

parallel or perpendicular index lines. Figure 37 shows some of the possible scale arrangements. Similarity of triangles is the only requisite.

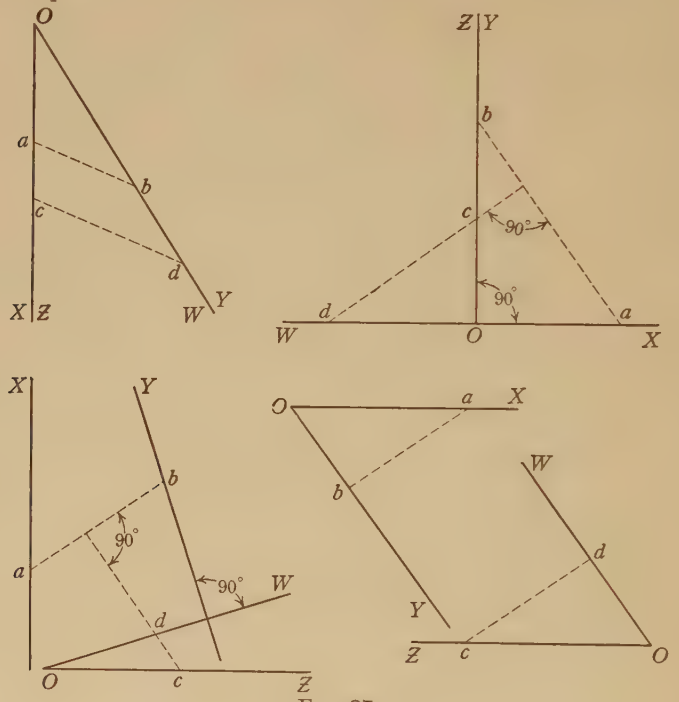


Fig. 37.

PROBLEMS

42. Construct a parallel scale alignment chart for the equation

$$P = 280TWL$$

T to vary from .01 to 0.10
W to vary from 1 to 10
L to vary from 1 to 10

43. Construct a parallel scale chart for the equation

$$H = \frac{D^2NS}{12}$$

D to vary from 3 to 6
S to vary from 3 to 8
and N to vary from 1 to 8 (integral values only).

44. Make an alignment chart for the equation

$$t = \frac{\sqrt[3]{n}}{S\sqrt[3]{2a}}$$

n varies from 12 to 64
S varies from 1 to 10
a varies from 6 to 12 (even numbers only).

45. Prepare a parallel scale chart for equation

$$I = K\sqrt{lL}$$

K to vary from 0.05 to 1
l and L to vary from 10 to 1000

46. Make a parallel scale chart for equation

$$V = 5.33cbH^{\frac{3}{2}}$$

c to vary from 0.50 to 0.70
b to vary from 2 to 20
H to vary from 0.1 to 1.6.

47. Construct a parallel scale chart for the equation

$$I = \frac{3.2}{10^3}\left(\frac{AT^2}{G}\right)$$

A to vary from 0.2 to 5
T to vary from 1000 to 30,000
G to vary from 0.05 to 1.0.

48. Construct an alignment chart for the equation

$$P = \frac{wh^2}{2}\left(\frac{1 - \sin\theta}{1 + \sin\theta}\right)$$

w to vary from 50 to 200
h to vary from 5 to 60
θ to vary from 0° to 60°.

49. Construct a parallel scale chart, to be read using a single index line for the equations

$$P_a = \sqrt[3]{P_1^2 P_2}$$

$$P_b = \sqrt[3]{P_1 P_2^2}$$

P_1 and P_2 to vary from 1 to 1000.

50. Construct a Z chart with diagonal graduated for the equation

$$M = 0.2FD^3$$

F to vary from 0 to 20,000
D to vary from 0 to 4.5.

51. Construct a parallel scale chart for equation

$$t = \frac{pr}{f \sin \theta}$$

p to vary from 25 to 200
r to vary from 6 to 45
f to vary from 1000 to 8000
θ to vary from 5° to 50°.

52. Construct a parallel scale chart for

$$n = \frac{CEd^4}{8PD^3}$$

C varies from 0.25 to 3
d varies from 0.1 to 0.6
P varies from 25 to 500
D varies from 1 to 4
$E = 12{,}000{,}000.$

53. Prepare an alignment chart for the equation

$$W = \frac{\pi bf}{P}\left(0.124 - \frac{0.684}{T}\right)$$

W to vary from 10 to 10,000
P to vary from 1 to 20
f to vary from 2000 to 20,000
T to vary from 12 to 100.

54. Construct a parallel scale alignment chart for

$$T = \frac{pr}{f \sin \theta}\left(\frac{1 - n^2}{n}\right)$$

n to vary from 0.1 to 0.5
Other variables as in Problem 51.

55. Prepare a parallel scale alignment chart for the equation

$$W = \frac{0.8d^2f}{1 + \dfrac{L^2}{750d^2}}$$

d to vary from 1 to 10
f to vary from 1 to 5
L to vary from 10 to 200

(*Suggestion.* — Separate the equation into the equations,

$$\frac{L^2}{750d^2} = Q$$

$$0.8d^2f = Q_2$$

and $$W = \frac{Q_2}{Q_1},$$

where $$Q_1 = Q + 1.$$

Show values of the variable d on the X scales of the first two equations. Use the same X scale for both, thus giving only one scale for d.)

56. Construct a Z chart having scales 6 inches long for the equation

$$\frac{t}{b} = \frac{1.6 f_t}{f_c}$$

t and b up to 1.5

f_t up to 120,000.

57. Construct a Z chart for equation

$$V = \frac{1728 W L^3}{192 E I}$$

$E = 30{,}000{,}000$

L varies from 0 to 35

V varies from 0 to 1.25

W varies from 0 to 250,000.

58. Prepare a Z chart for the equation

$$a = \sqrt{\frac{4.5 f t^2}{p}}$$

p to vary from 0 to 250

f to vary from 0 to 8000

t to vary from 0 to 1.

59. Construct an alignment chart of the type shown in Fig. 36, for the equation

$$P = \frac{B^2 L}{C^2 w d^5}$$

		B	L	P	w	d
Limits,	0 to	400	1500	25	10	10

60. Construct an alignment chart combining two Z charts (see Fig. 36) for the equation

$$t = \frac{prF}{fe}$$

		p	r	F	f	e
Limits,	0 to	250	60	10	120,000	1.00

61. Construct an alignment chart for

$$p_1 V_1^n = p_2 V_2^n$$

p_1 and p_2 to vary from 1 to 500
V_1 and V_2 to vary from 2 to 50
n to vary from 1 to 2 by tenths.

(*Suggestion.* — Write equations

$$\frac{p_1}{p_2} = p$$
$$\frac{V_2}{V_1} = V$$
$$p = V^n.$$

Scales for p_2 and V_2 are inverted.)

62. Chart the equation

$$U = \frac{200\sqrt{V}}{r \log_e \left(\frac{rq}{rq - 950} \right)}$$

Limits, V, 0 to 10
r, 25 to 120
q, 10 to 50.

CHAPTER IV

MISCELLANEOUS FORMS

ALIGNMENT CHARTS for certain equations which occur less frequently in engineering practice will be presented in this chapter.

Form $x + y = \frac{z}{w}$. The first case is that of equations taking the form

$$x + y = \frac{z}{w}.$$

The chart for this equation resembles the Z chart. As shown in Fig. 38 parallel index lines are used. The equation is satisfied for values of the variables cut by an index line from X to Y, and a parallel index line from Z to W. Such index lines are ab and cd (Fig. 38). The diagonal OO_1 is given a definite length, K.

To determine the necessary relation of the scale moduli, assume specific values for x, y, z, and w which satisfy the equation. Let these values be a, b, c, and d, respectively. Then

$$a + b = \frac{c}{d}.$$

Draw O_1r parallel to ab, then from similar triangles, OrO_1, and Ocd,

$$\frac{M_x a + M_y b}{K} = \frac{M_z c}{M_w d}.$$

If the X and Y scales have equal moduli, viz., $M_x = M_y$

$$\frac{M_x(a + b)}{K} = \frac{M_z c}{M_w d}$$

or, since

$$a + b = \frac{c}{d}$$

$$M_w = \frac{KM_z}{M_x}. \tag{4}$$

It is evident that limits can be assigned to only two variables.

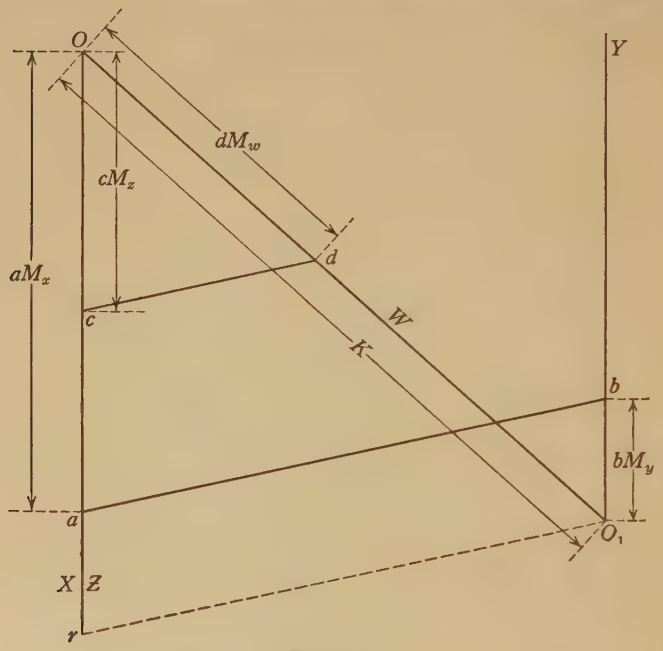


Fig. 38.

Example 10. Prepare an alignment chart for the equation

$$I = \frac{h^3}{12}(b + 3d)$$

given the following limits

b, 0 to 14
h, 0 to 5.

Limits are specified for only two of the variables as the scale moduli for the X and Y scales are the same.

Rewrite the equation, as

$$b + 3d = \frac{12I}{h^3}.$$

This equation is in the form, $x + y = \frac{z}{w}$,

where

$$x = b,\ y = 3d,\ z = 12I, \text{ and } w = h^3.$$

From the limits given for b,

$$M_x = M_b = 0.5 \text{ for a 7-inch scale length.}$$

Since $M_x = M_y$

$$M_y = 0.5 \quad \text{and} \quad M_d = 3 \times 0.5 = 1.5.$$

The scale modulus $M_w = M_{h^3}$ is found from the limits for h,

h varies from 0 to 5

$w = h^3$ varies from 0 to 125

$M_w = \frac{5}{125} = 0.04$ for a 5-inch scale length

$M_{h^3} = 0.04.$

The scale modulus, M_z, is obtained from equation (4), assuming $K = 10$ ins.

$$0.04 = \frac{10M_z}{0.5}$$

$$M_z = 0.002$$

or $$M_I = 12M_z = 0.024.$$

The completed chart is shown in Fig. 39.

The b (X) scale may be graduated conveniently with an engineer's scale having 20 divisions to the inch. The d (Y) scale is graduated uniformly with 1.5 inches showing unity increase of the variable. The I (Z) scale is graduated uniformly with a scale 2.4 inches long for each 100 increase in the variable, as

$M_I = 0.024$, viz., 0.024 inch shows unit increase in I. The h (W) scale is a non-uniform scale since the variable appears in the equation as a cube. The scale of h^3, however, is a uniform

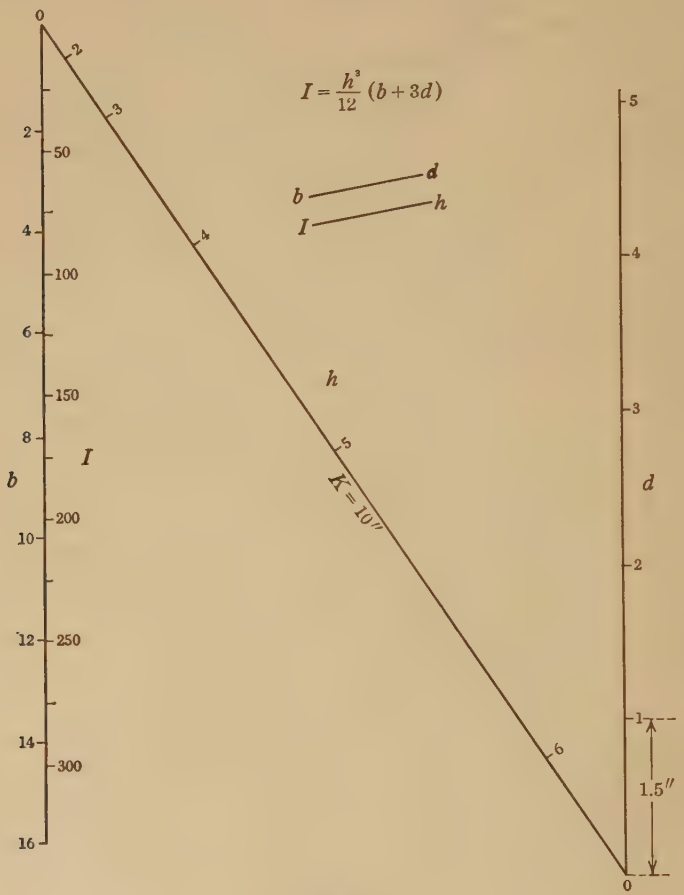


Fig. 39.

scale with a scale modulus, $M_{h^3} = 0.04$. Therefore a uniform scale 4 inches long divided into 100 parts may be used to graduate the h scale. Values of h^3 may be laid off in accordance with the

following table and the corresponding points charted in values of h.

h	0	1	2	3	4	5	6
h^3	0	1	8	27	64	125	216

For equations of the form

$$x - y = \frac{z}{w}$$

positive values of y are laid off on the other side of O_1 (Fig. 38) as shown in Fig. 40, the negative sign merely inverting the scale starting with the zero value at O_1. Otherwise the negative sign does not affect the solution. The W scale may, of course, be graduated beyond the point O_1.

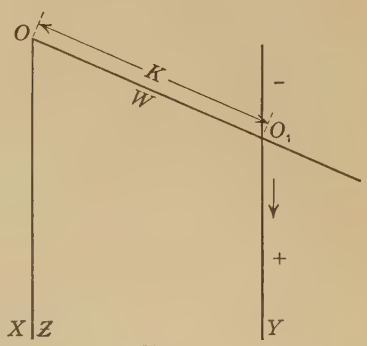


FIG. 40.

Form $\frac{x+y}{x-y} = \frac{z}{w}$. The next case is that of equations taking the form

$$\frac{x+y}{x-y} = \frac{z}{w}.$$

For this case the scales are laid off as shown in Fig. 41. The angle between the X and Z scales is 45 degrees. The X and Y scales are perpendicular, as are the Z and W scales. The index lines ab and cd are parallel.

To determine the necessary relation of the scale moduli assume specific values of a, b, c, and d, for x, y, z, and w, respectively.

Then

$$\frac{a+b}{a-b} = \frac{c}{d}.$$

The several distances for these points along the scales are shown in Fig. 41. With O as a center describe arc fbg with radius equal to M_yb. Draw fb and bg. Draw gh parallel to fb.

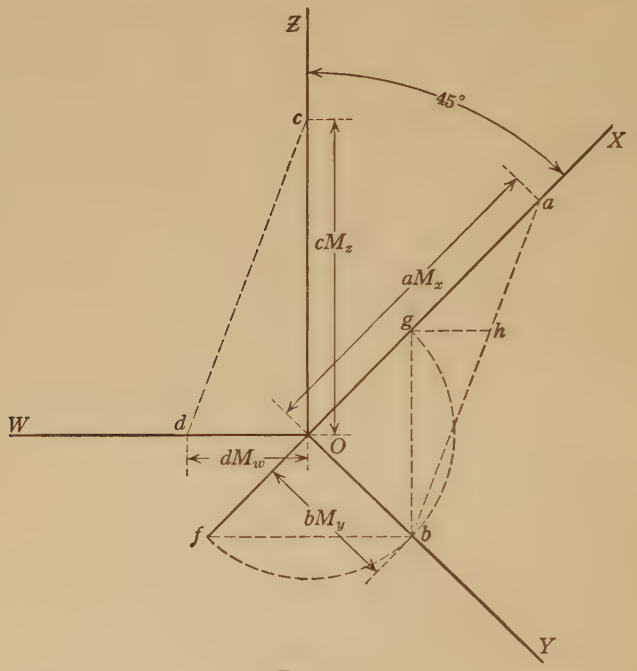


Fig. 41.

Then, since $Of = Og = M_yb$, from similar triangles afb and agh,

$$\frac{af}{ag} = \frac{fb}{gh} = \frac{M_xa + M_yb}{M_xa - M_yb}.$$

Also

$$\frac{fb}{gh} = \frac{gb}{gh} = \frac{M_zc}{M_wd},$$

since fb equals gb, and triangle dOc is similar to hgb.

Therefore,

$$\frac{M_x a + M_y b}{M_x a - M_y b} = \frac{M_z c}{M_w d}.$$

Using the same scale modulus for x and y,

$$\frac{M_x(a+b)}{M_x(a-b)} = \frac{M_z c}{M_w d},$$

or $$1 = \frac{M_z}{M_w} \quad \text{since} \quad \frac{a+b}{a-b} = \frac{c}{d}$$

hence $$M_z = M_w.$$

It follows that only two different scale moduli are used, one for the X and Y scales, and one for the Z and W scales.

Example 11. Construct a chart for the equation

$$\frac{f+p}{f-p} = \frac{D^2}{d^2}.$$

Limits to be

f, 0 to 4000
D, 0 to 10.

Determine scale moduli as follows:

$$M_x = M_f = \tfrac{1}{1000} = 0.001 \text{ (4-inch scale length).}$$

Since $M_x = M_y$

$$M_y = M_p = 0.001$$

$$M_z = M_{D^2} = \tfrac{5}{100} = 0.05 \text{ (5-inch scale length).}$$

Since $M_w = M_z$

$$M_w = M_{d^2} = 0.05.$$

The chart is shown in Fig. 42. The f and p scales are graduated uniformly, each inch of length showing an increase of 1000 units. The D and d scales are graduated with uniform scales for the squares of the variables, five inches of length being an increase of 100 units, the points 4, 9, 16, 25, etc. of the scale being given the values 2, 3, 4, 5, etc., respectively.

Another arrangement of scales is obtained by revolving the Z and W scales (Fig. 41) as a unit ninety degrees clockwise. The index lines are then perpendicular instead of parallel.

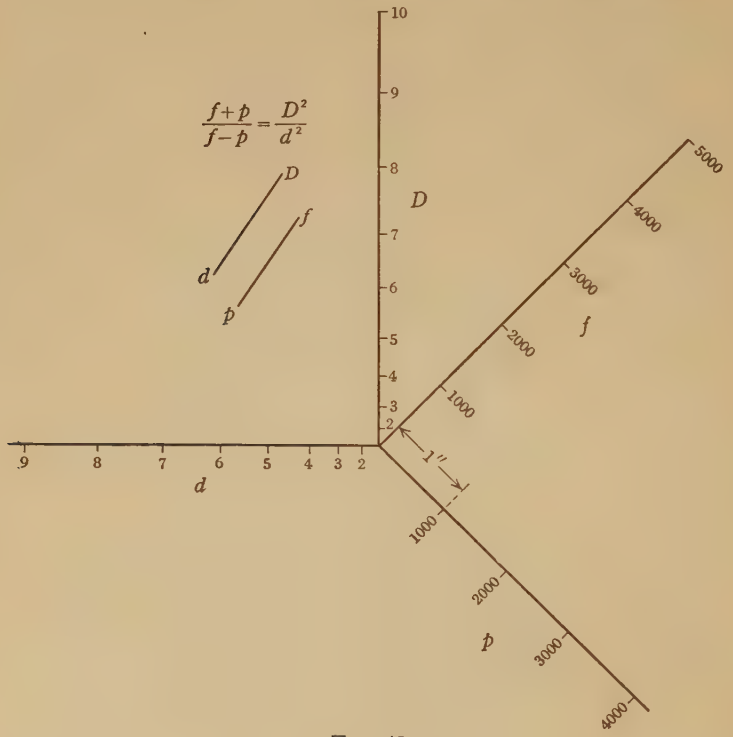


Fig. 42.

Form $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$. Equations of the form

$$\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$$

may be considered as of the form $x + y = z$, but as this requires the use of reciprocal scales which, in general, are not satisfactory, it is better practice to use a different type of chart. Assume three scales radiating from the point O (Fig. 43). If the outside

scales are graduated with values of the variables x and y, and the intermediate scale with values of z, a straight line (acb) will cut the scales in values which satisfy the equation. To determine the relation between angles α and β, and the necessary scale modulus for the z scale, draw construction line rc parallel to the

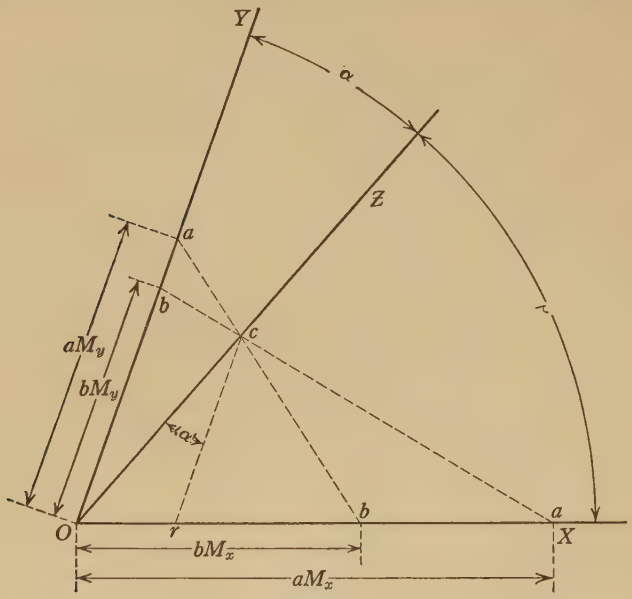


FIG. 43.

Y scale, and assume a, b, and c as specific satisfying values for the variables x, y, and z, respectively.

Then
$$\frac{1}{a}+\frac{1}{b}=\frac{1}{c},$$

that is
$$c=\frac{ab}{a+b}.$$

The index lines, $x = a$, $y = b$, and $x = b$, $y = a$, must intersect the z scale at the same point, viz., $z = c$, since the value of z is the same for both pairs of values for x and y.

From similar triangles rca and Oba, (a on X scale, b on Y scale)

$$\frac{M_x a - Or}{M_x a} = \frac{cr}{M_y b},$$

and from similar triangles rcb and Oab, (b on X scale, a on Y scale)

$$\frac{M_x b - Or}{M_x b} = \frac{cr}{M_y a}.$$

Rewriting these equations

$$1 - \frac{Or}{M_x a} = \frac{cr}{M_y b},$$

and

$$1 - \frac{Or}{M_x b} = \frac{cr}{M_y a}.$$

Subtracting

$$\frac{Or}{M_x}\left(\frac{1}{b} - \frac{1}{a}\right) = \frac{cr}{M_y}\left(\frac{1}{b} - \frac{1}{a}\right),$$

hence

$$\frac{M_x}{M_y} = \frac{Or}{cr} = \frac{\sin \alpha}{\sin \beta}.$$

Thus, having determined the scale moduli, M_x and M_y, one angle may be assumed and the other found.

To find the scale modulus for the Z scale, M_z, note that $Oc = M_z c$.

Then

$$\frac{M_z c}{Or} = \frac{\sin (180° - (\alpha + \beta))}{\sin \alpha}$$

and

$$M_z c = Or\left(\frac{\sin (\alpha + \beta)}{\sin \alpha}\right).$$

To find the value of Or combine the following two equations previously found, viz.,

$$1 - \frac{Or}{M_x a} = \frac{cr}{M_y b},$$

and

$$\frac{M_x}{M_y} = \frac{Or}{cr},$$

obtaining $$1 - \frac{Or}{M_x a} = \frac{M_y(Or)}{M_x M_y b} = \frac{Or}{M_x b},$$

whence $$M_x ab = Or(a + b)$$

and $$Or = \frac{M_x ab}{a + b} = M_x c,$$

since $$c = \frac{ab}{a + b}.$$

Substituting for Or, $$M_z = \frac{M_x \sin(\alpha + \beta)}{\sin \alpha},$$

an expression which may be used in determining the scale modulus M_z.

For $M_x = M_y$

$$\sin \alpha = \sin \beta,$$

and α may be made equal to β.

Then $M_z = M_x \frac{\sin 2\alpha}{\sin \alpha} = M_x (2 \cos \alpha)$.

For $M_x = M_y = M_z$

$$\alpha = \beta = 60°.$$

An example illustrating this last condition follows: —

Example 12. Chart the equation

$$\frac{1}{R} = \frac{1}{V_1} + \frac{1}{V_2}$$

R to vary from 0 to 10.

Transpose the equation as

$$\frac{1}{V_1} + \frac{1}{V_2} = \frac{1}{R}.$$

Then $x = V_1$, $y = V_2$ and $z = R$.

All scales are uniform, and are graduated using the same modulus, viz., $M_x = M_y = M_z = 0.5$.

Scale length is therefore 5 inches, and $M_{V_1} = M_{V_2} = M_R = 0.5$. Figure 44 shows the completed chart.

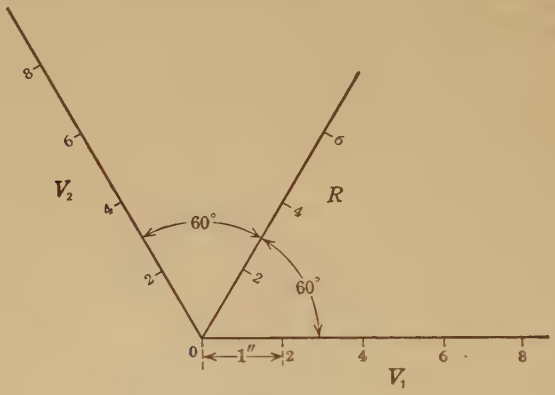


FIG. 44.

Alignment charts of this type may be made to include more than three variables if of the form

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}+\text{etc.}=\frac{1}{w},$$

by using ungraduated turning scales. Write the equation as

$$\frac{1}{x}+\frac{1}{y}=\frac{1}{Q},$$

and

$$\frac{1}{Q}+\frac{1}{z}=\frac{1}{w},$$

and proceed to construct two, three variable charts with one scale (Q) common to both.

Form $x + z^n \cdot y = z^{n_1}$. Alignment charts may be constructed for equations of the form

$$x + z^n \cdot y = z^{n_1}$$

where the variable z is raised to the different powers n and n_1 both of which are constants.

Such equations may be charted on a three-scale chart, the outer (X and Y) scales being straight and parallel, while the intermediate (Z) scale is curved. The position of the Z scale is determined by passing a smooth curve through a series of points found by assigning specific values for the variable z. Each point so found will carry the value of z used in locating that point. In determining the points of the Z scale it will be necessary to deduce two formulas, one of which will locate the Z scale and the other the position of the point on the Z scale. The first formula will contain the value of the z variable represented by z^n of the general formula, while the second formula will be in terms of both values of the z variable. Moreover, since the z modulus will vary for each specific value of z, such as $z = c$, the second formula for the location of the point $z = c$ will be found as a value for $M_z c$. This will definitely locate the point from the zero line on the position of the Z scale for $z = c$, found by the first formula.

To deduce these formulas, let the variables have the following specific values, which will satisfy the general equation.

$$x = a, \quad y = b, \quad \text{and} \quad z = c.$$

Then substituting

$$a + c^n b = c^{n_1}$$

from which equation the location on the Z scale of the point $z = c$ is to be found.

In Fig. 45, let the distance between the X and Y scales be $r + s$, and the scale moduli for these scales be M_x and M_y, respectively.

Draw line $x = 0$, $y = 0$. The value $z = 0$ must always lie on this line, viz., at a point such as O', and values of z will be measured from this line, as $M_z c$.

Draw line $x = a$, $y = b$. This line must intersect the Z scale at point c, since the values, $x = a$, $y = b$, and $z = c$ satisfy the equation.

If $x = c^n b$, $y = \frac{a}{c^n}$ and $z = c$, the equation will be satisfied, since by substituting these values in the general equation there results the equation

$$c^n b + a = c^{n_1}$$

known to be a true equation. Therefore a line from point

$$x = c^n b \quad \text{to} \quad y = \frac{a}{c^n}$$

must also pass through point c on the Z scale.

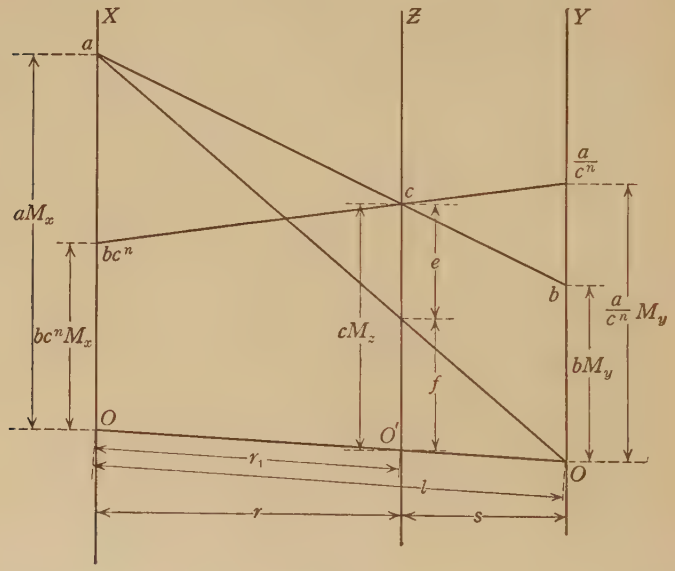


Fig. 45.

In the similar triangles with common apex at c

$$\frac{M_x a - M_x c^n b}{M_y \frac{a}{c^n} - M_y b} = \frac{r}{s}$$

$$\frac{M_x(a - c^n b)}{M_y\left(\frac{a - bc^n}{c^n}\right)} = \frac{r}{s}$$

$$\frac{M_x c^n}{M_y} = \frac{r}{s}$$

or
$$\frac{M_x c^n}{M_x c^n + M_y} = \frac{r}{r + s} \tag{5a}$$

or
$$\frac{M_y}{M_x c^n + M_y} = \frac{s}{r + s}. \tag{5b}$$

Either one of the equations (5) may be used to determine the position of the Z scale which carries the c value of z.

For convenience in laying off, equation (5a) may be written

$$r_1 = \frac{c^n M_x l}{c^n M_x + M_y} \tag{6}$$

where r_1 and l are measured along the line $x = 0$, $y = 0$. This distance will always be less than l for positive values of c.

Having determined the position of the Z scale, the distance of the point c, $(M_z c)$ above the line $x = 0$, $y = 0$ may be found.

Draw the line $x = a$, $y = 0$.

Then $M_z c = e + f$

$M_z c = M_y b\left(\frac{r}{r + s}\right) + M_x a\left(\frac{s}{r + s}\right)$. From similar triangles

$M_z c = M_y b\left(\frac{M_x c^n}{M_x c^n + M_y}\right) + M_x a\left(\frac{M_y}{M_x c^n + M_y}\right)$. From eq's. (5)

$$M_z c = \frac{M_x M_y}{M_x c^n + M_y}(bc^n + a),$$

or, since $bc^n + a = c^{n_1}$

$$M_z c = \frac{M_x M_y c^{n_1}}{M_x c^n + M_y}. \tag{7}$$

Equation (7) gives the distance on the Z scale in terms of the functions of z, viz., c^n and c^{n_1}.

Since for a given positive value of z, there can be but one position of the scale, the graduations for this scale may be carried on the line $x = 0$, $y = 0$.

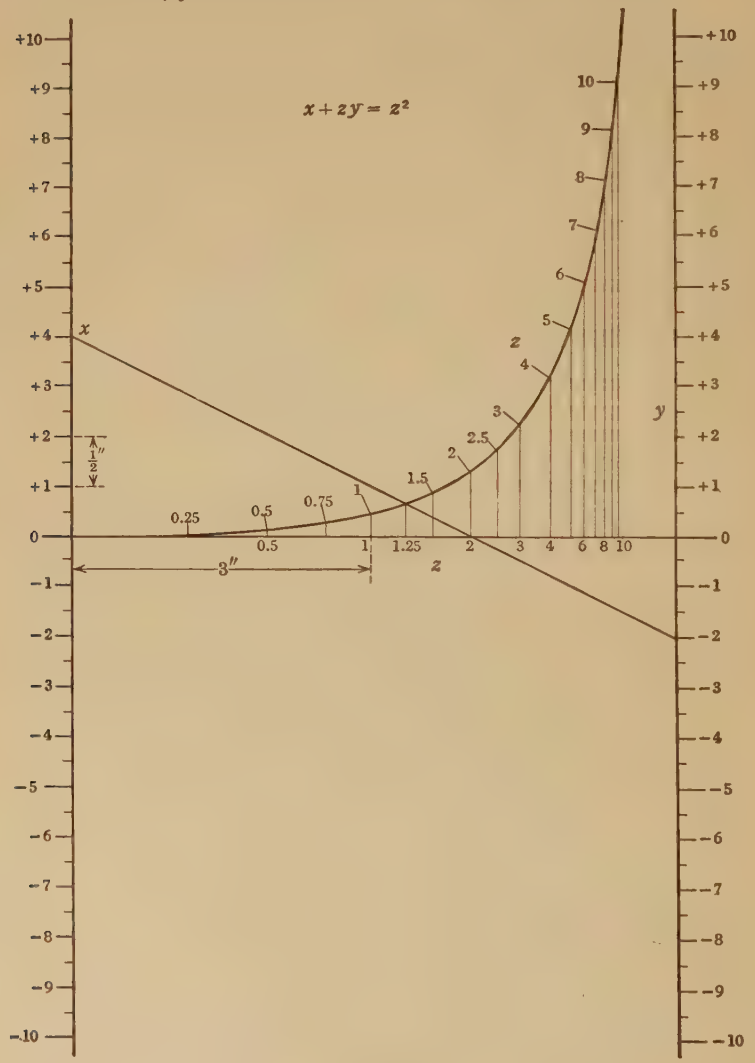


Fig. 46.

Figure 46 shows the chart for the solution (positive roots only) of the quadratic equation

$$x + z \cdot y = z^2.$$

x and y to vary from -10 to $+10$.

Assuming $M_x = M_y = 0.5$ and $l = 6$ ins. the position of the Z scale for $z = 1$ ($c = 1$) is found from equation (6), using $n = 1$, and $n_1 = 2$,

$$r_1 = \frac{1^1 \times 0.5 \times 6}{1^1 \times 0.5 + 0.5} = 3 \text{ ins.}$$

and the distance on this scale to point $z = 1$ is found from equation (7)

$$M_z C = \frac{0.5 \times 0.5 \times 1^2}{1^1 \times 0.5 + 0.5} = \tfrac{1}{4} \text{ in.}$$

Other values of z may be found from values given in the following table.

z	r_1	$M_z C$
$\frac{1}{4}$	1.20	0.025
$\frac{1}{2}$	2.00	0.08
1	3.00	0.25
$1\frac{1}{2}$	3.60	0.45
2	4.00	0.67
3	4.50	1.13
4	4.80	1.60
5	5.00	2.08
10	5.45	4.55

The index line shows that for

$$x = 4 \quad \text{and} \quad y = -2$$
$$z = 1.25.$$

Negative values of z continue the curve to the left of the X scale and above the $x = 0$, $y = 0$ line, and to the right of the Y scale below the $x = 0$, $y = 0$ line.

An interesting modification of this case occurs when the curved scale carrying values of the two-function variable is placed in the X scale position. The general equation may then be written in the form

$$x^{n_1} + y = x^n \cdot z.$$

This involves merely a transposition of terms from the former case, the change in variables being made to keep the scales in their

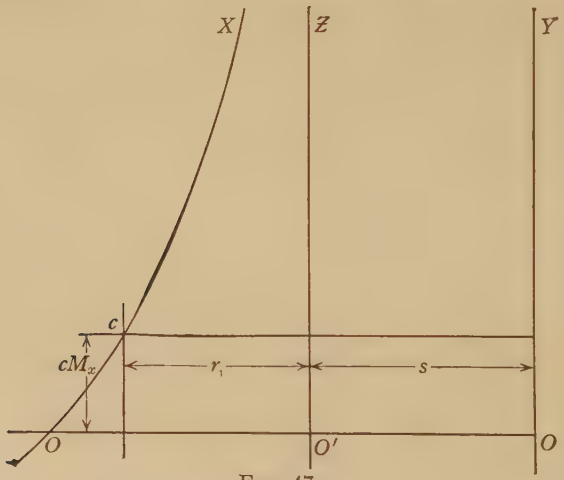


FIG. 47.

customary relative positions (Fig. 47). The distance between the Z and Y scales, s, is constant. The variable distance r_1, locating the position of the X scale for any given value of x, as $x = c$, may be obtained from the equation

$$r_1 = \frac{M_z s}{M_y c^n - M_z}.$$

Limits having been imposed upon the y and z variables their scale moduli, M_y and M_z, are known quantities. The distance

along the X scale, so determined, from the zero line to the point c, may be shown to be

$$M_x c = \frac{M_z M_y c^{n_1}}{M_y c^n - M_z}.$$

Thus, by giving x (c) different values the position and graduations of the X scale may be found.

The method of procedure for such a case is outlined under Problem 73, at the end of this chapter.

PROBLEMS

63. Prepare a chart for the equation

$$I = \frac{W}{12}(3r^2 + h^2)$$

W to vary from 0 to 100
r to vary from 0 to 9
h to vary from 0 to 15.

Show index lines to solve, $r = 8$, $h = 12$, $W = 40$, $I = ?$

64. Prepare a chart for the equation

$$S = \frac{W}{W - W_1}$$

W to vary from 0 to 100.

(The X and Z scales (Fig. 40) are identical and only one index line is used.)

65. Prepare a chart for the equation

$$(T_1 - T_2)V = 33{,}000H$$

T_1 and T_2 to vary from 0 to 1000
V to vary from 0 to 3000.

66. Make a chart for the equation

$$E = \frac{W}{64}(V_1^2 - V_2^2)$$

Limits, W, 0 to 100
V_1 and V_2, 0 to 50.

67. Construct a chart for the equation

$$\frac{f}{P} = \frac{R_1^2 + R_2^2}{R_1^2 - R_2^2}$$

f to vary from 0 to 5000
R_1 to vary from 0 to 10.

68. Prepare an alignment chart for the equation

$$\frac{1}{w^2} = \frac{1}{(\pi P)^2} + \frac{1}{L^2}$$

Limits, for P, 0 to 2.5
for L, 0 to 3.

69. Construct an alignment chart for the equation

$$\frac{1}{N^2} = \frac{1}{N_1^2} + \frac{1}{N_2^2} + \frac{1}{N_3^2}$$

N_1, N_2 and N_3 to vary from 0 to 100.

70. Make an alignment chart for the equation

$$P - q^{\frac{1}{2}}N = q$$

P and N to vary from 0 to 5.

(Note, the minus sign inverts the N scale.)

71. Prepare an alignment chart for the equation

$$S = V_0 t - 16t^2$$

S to vary from 0 to 320
V_0 to vary from 0 to 200.

(*Suggestion.* — Write the equation as

$$-\frac{S}{16} + \frac{V_0}{16} \cdot t = t^2.\Big)$$

72. Prepare a chart for the solution of the cubic equation

$$h^3 + c \cdot h + d = 0$$

c and d to vary from -10 to 10.

73. Prepare an alignment chart for the equation

$$f = \frac{R_1^2 - R_2^2}{R_1 - R_2}$$

R_1 and R_2 to vary from 4 to 30
f to vary from 10 to 60.

(*Suggestion.* — Rewrite the equation as

$$R_1^2 - R_2^2 + fR_2 = fR_1.$$

Then let, $$Q = fR_2 - R_2^2.$$

Hence $$R_1^2 + Q = fR_1,$$

and $$R_2^2 + Q = fR_2.$$

The last two equations are in the form

$$x^{n_1} + y = z \cdot x^n,$$

where

$$x = R_1 \text{ or } R_2$$
$$y = Q$$
$$z = f$$
$$n_1 = 2$$
$$n = 1.$$

From the limits for f, determine $M_z = M_f$.

Assume $M_y = M_Q = 0.1$ and s (Fig. 47) $= 3$ ins.

The X scales, graduated for R_1 and R_2 are identical.

The Z scale, carries values of f.

The Y scale is a turning scale and does not appear on the chart, a single index line cutting the X scale at two points gives values for R_1, R_2 and f which satisfy the equation.)

74. Chart the equation

$$y = a\left[e^{c \cdot \frac{v^2}{\sigma}\left(1-\frac{x^2}{r^2}\right)}\right]$$

Limits, a, 1 to 10

V, 100 to 600

σ, 1000 to 6000

$\frac{x}{r}$, 0 to 0.95

$c = 4 \cdot 10^{-6}$.

75. Construct an alignment chart for the graphical determination of the positive roots of h in the cubic equation,

$$h^3 + lh + d = 0$$

l and d to vary from -10 to $+10$.

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